From dust to planets in a turbulent nebula

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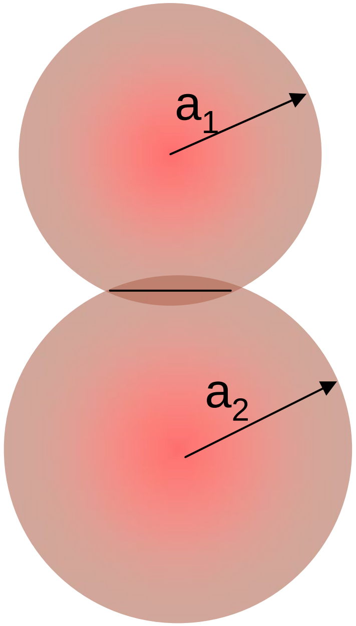
Summary

- **Grain sticking and aggregate formation**
  - Models and experiments
- **Gas acting on particles**
  - Drag force
  - Effects in a laminar disk: sedimentation and radial motion
- **Gravitational collective effects in the dust sub-disk**
- **Turbulent diffusivity**
  - Global turbulence
  - Feedback of particles over gas (Kelvin-Helmoltz instability)
- **What is turbulence after all?**
  - 3D and 2D
  - Effects of rotation
- **Turbulent, not random – the danger of simplicity**
  - Structures in a turbulent flow: effects over particles
  - Are disks globally turbulent? Sources of turbulence
- **Extreme exemples: stability and its consequences**

Grain glue, aggregate structure



- Van der Waals interactions (induced dielectric forces).

  For small, hard grains (Derjaguin *et al.* 1975):

  $$F_c = 4\pi\gamma_s R, \qquad R = a_1 a_2 / (a_1 + a_2)$$

  Confirmed with SiO_2 spheres, R~0.5-2.5 μm (Heim *et al* 1999).

- Electro-static interactions (10^3 x stronger)

- Rolling-friction forces (reshaping, compaction, energy absorption)

  ~10^{-10} N (Heim *et al.* 1999).

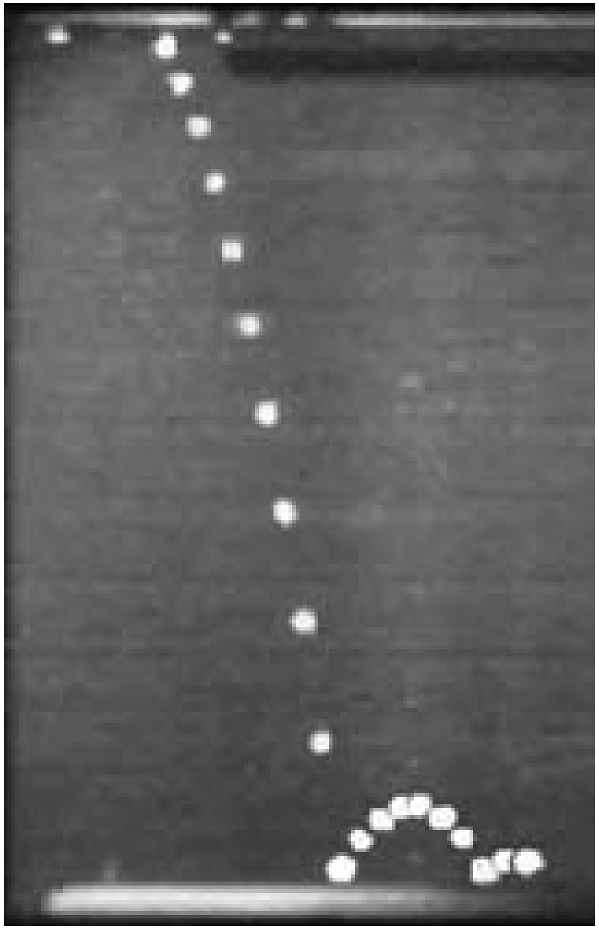
  Observed under scanning electron microscope (Heim *et al.* 2005)

Other glues: ices

- Problem: rest. coeff. ~80%

- Rest. coeff. 8% if <40 ^oK
- Electric charges important
- Polarization possible by mutual collision

Wang et al. 2005

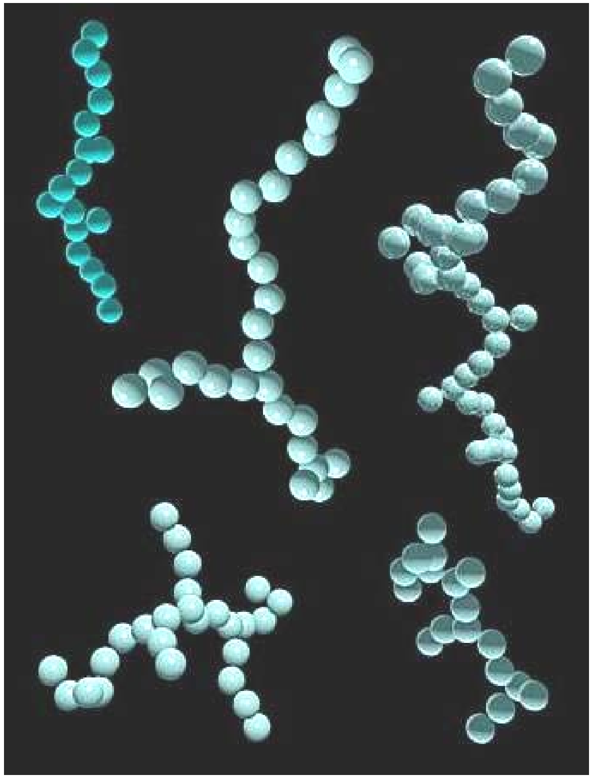


Grain-grain collisions

- Spherical grains:
  - Poppe et al (2000) experiments (« hard » SiO_2 grains)
    - Sharp transition from sticking to bouncing for v>1-2 m/s
    - Ave. restitution coeff. decreasing with v (large scatter)
    - No theoretical models available for this threshold
  - Models (« soft » polystyrene grains)
    - restitution coeff. increasing with v (Bridges et al. 1996)
    - Theory available (Chosky et al 1993): sticking v<<1 m/s
- Irregular grains:

  Smooth transition: even at v~100 m/s sticking is marginally possible.

Aggregates: fractal particles



$$m(a) \propto a^{D_f}$$

- Mono-size experiments:
  - Df~1.4 brownian motion
  - Df~1.9 turbulence
  - Df~1.8 sedimentation
- Models
  - Numerical studies (Kempf et al. 1999)
    - $\langle m \rangle \sim t^k$ ok with experiments
    - Lower Df→1 for low mean free path (random walk)

Grain growth: Smoluchowski's equation

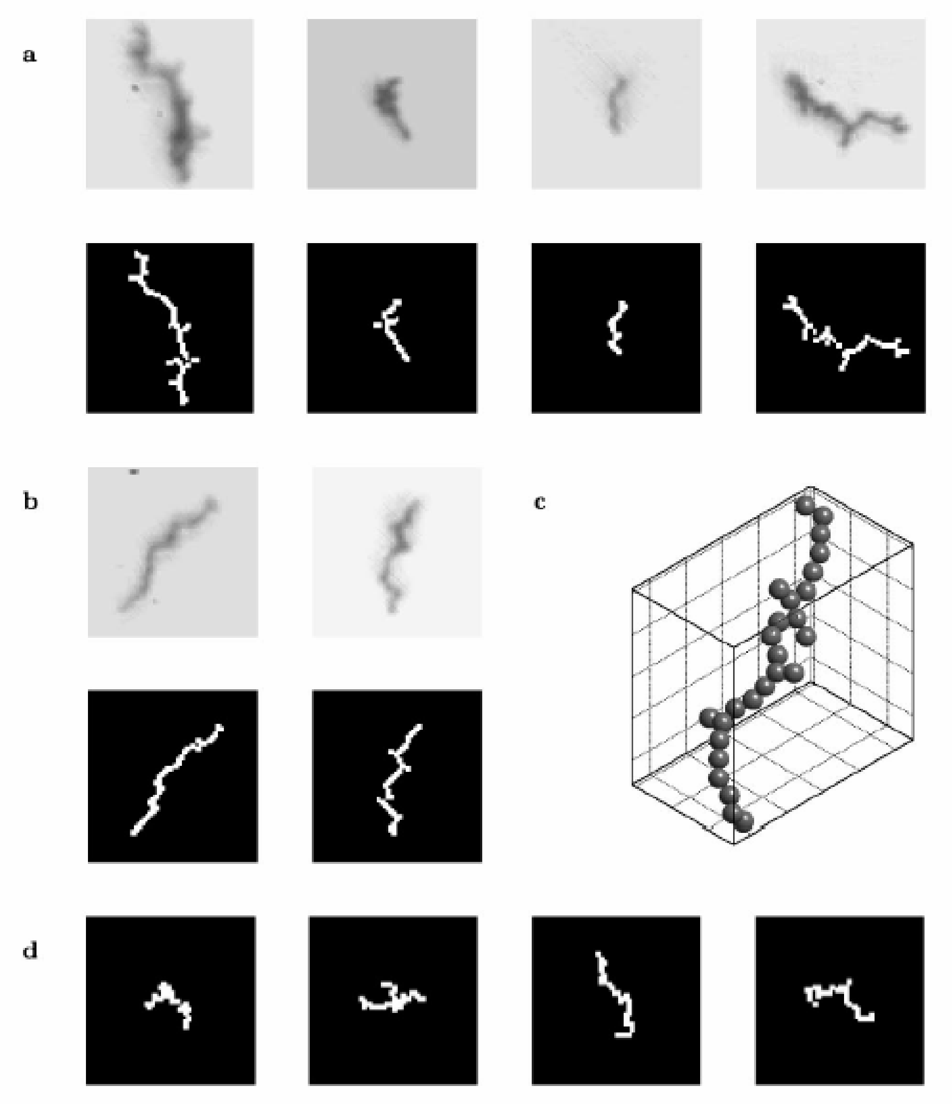
$$\begin{aligned}\frac{\partial n(m,t)}{\partial t} \quad = \quad & \frac{1}{2}\int_0^m K(m', m-m') \\ & \cdot n(m',t)\, n(m-m',t)\,\mathrm{d}m' \\ & - n(m,t)\int_0^\infty K(m',m)\, n(m',t)\,\mathrm{d}m' \,.\end{aligned}$$

The most used theoretical model for growth prediction.

The formulation of the kernel K is crucial.

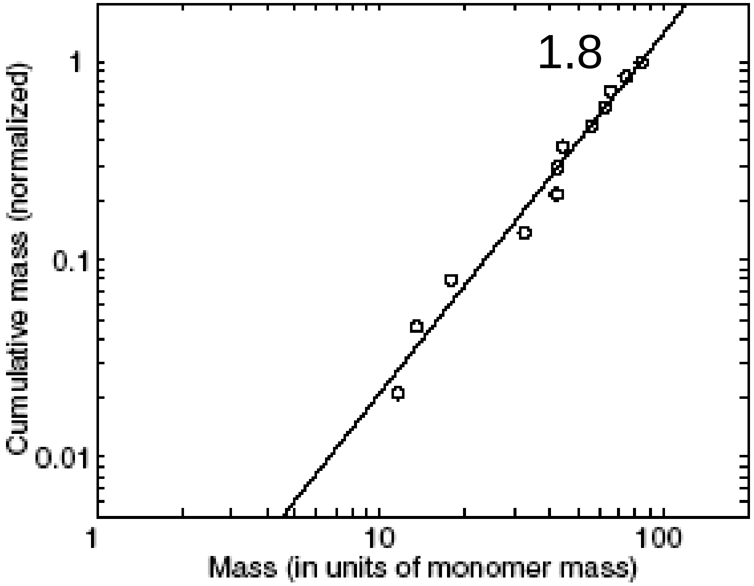
Some successfull predictions (Wurm and Blum 1998)

Micro-gravity experiments



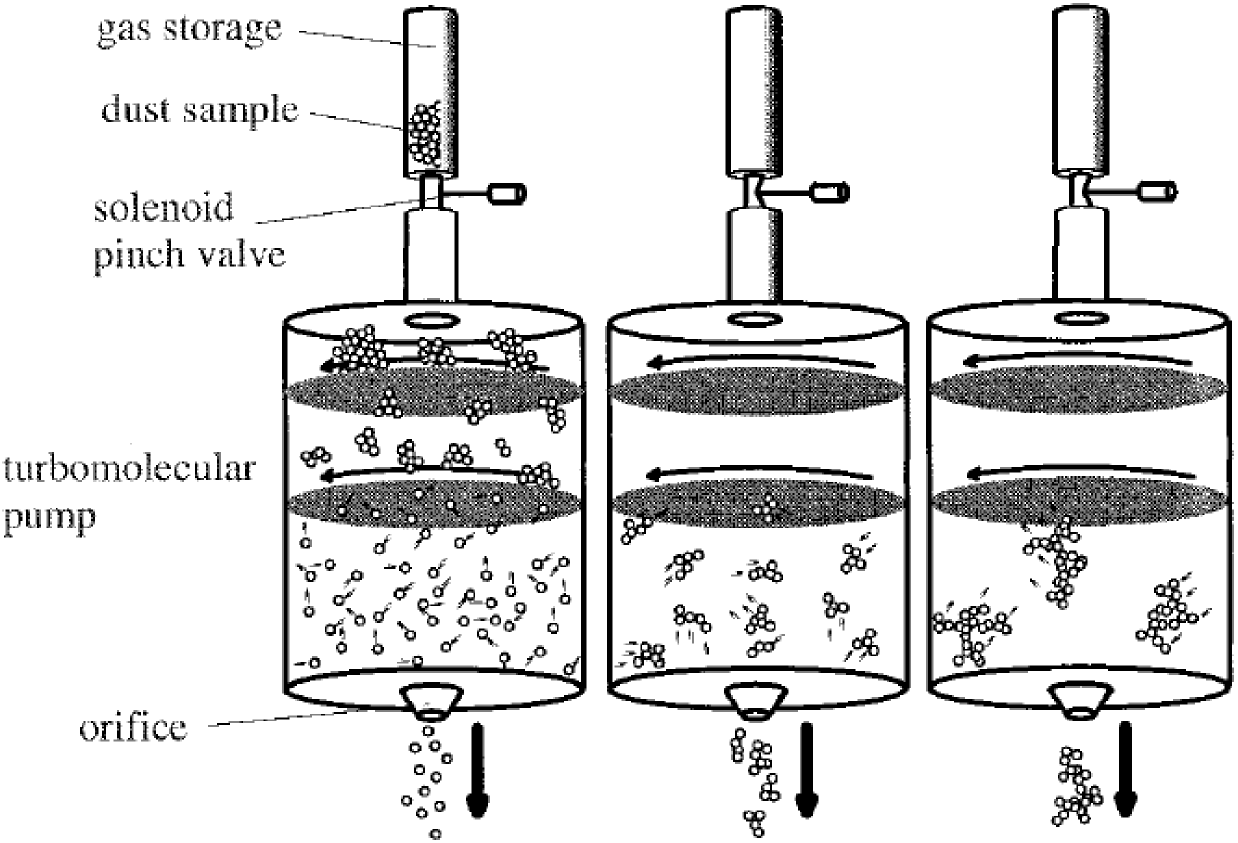
CODAG experiment

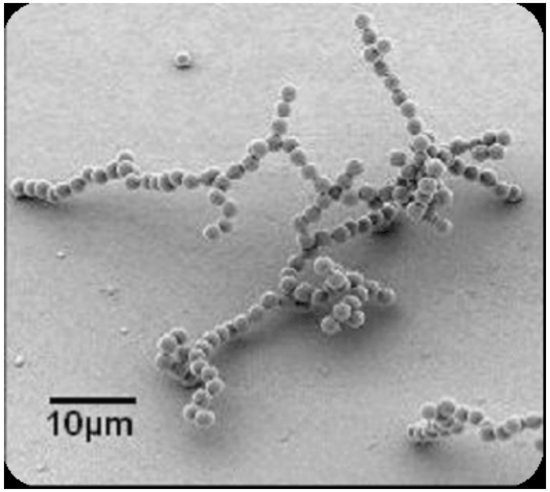
(Cosmic dust aggregation)



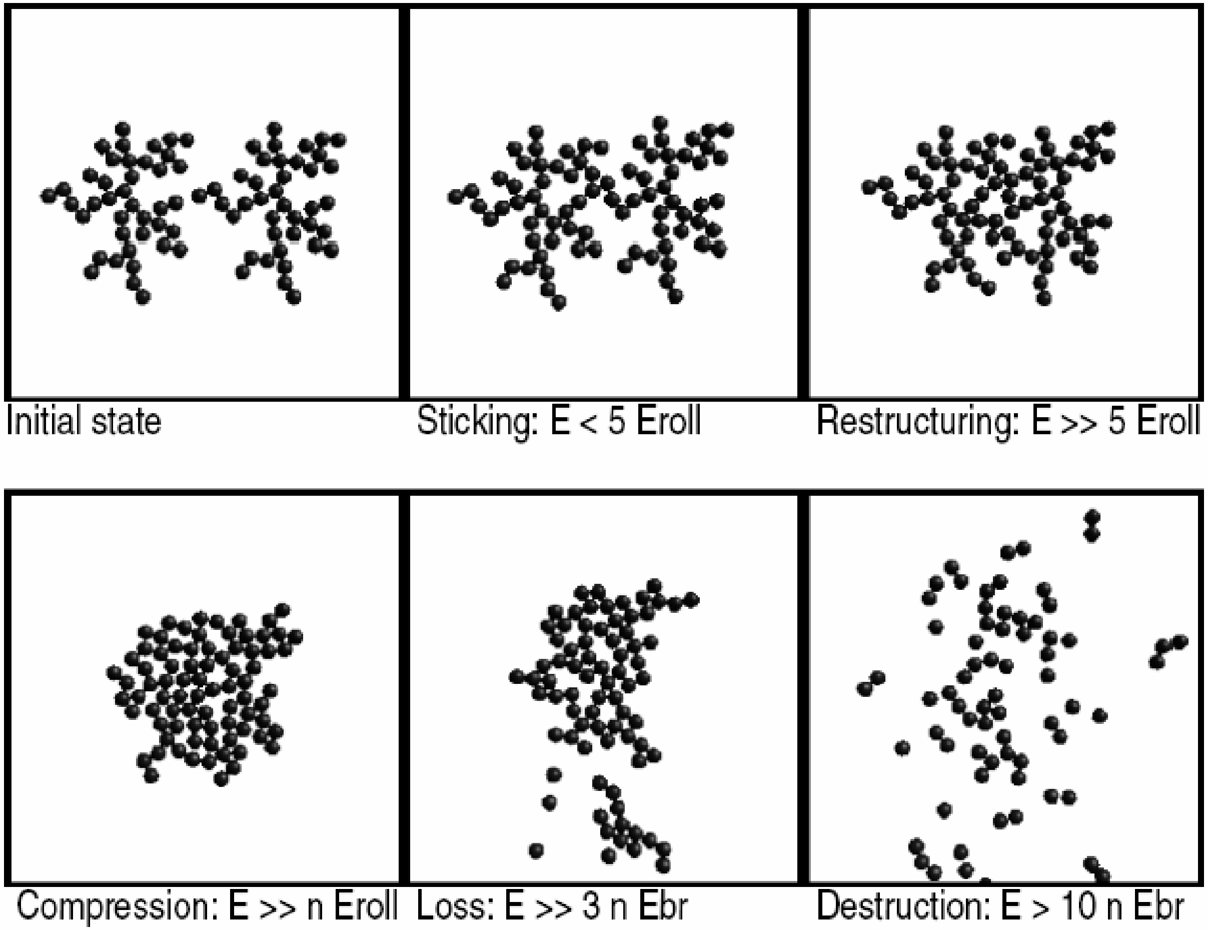
Blum et al. 2000

Laboratory experiments





Aggregate-aggregate collisions: results



Initial state

Sticking: E < 5 Eroll

Restructuring: E >> 5 Eroll

Compression: E >> n Eroll

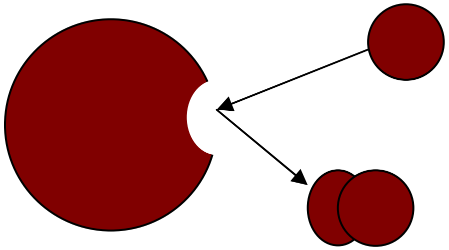
Loss: E >> 3 n Ebr

Destruction: E > 10 n Ebr

Dominik, Tielens (1997) – Wurm, Blum (2000)

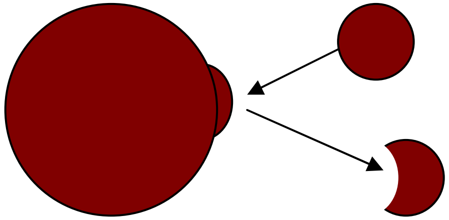
Macroscopic aggregates

mm vs. cm particles



Low-velocity impacts:

Sticking up to 1-2 m/s
Mass transfer to the projectile



High-velocity impacts:

No sticking
Mass transfer (1/2) to the target for V>13 m/s
(Wurm *et al.* 2005)

Small-size particles
are kept aboundant
in the disk

No satisfying theoretical model.

Gas coupling: drag on a sphere

« Equation of motion for a small rigid sphere in a nonuniform flow »:
Maxey and Riley (1983)

$$\rho_p \frac{d^2\mathbf{x}}{dt^2} = \rho_f \frac{D^2\mathbf{u}}{Dt^2} \boxed{-\frac{9\mu}{2a^2}\left(\frac{d\mathbf{x}}{dt}-\mathbf{u}-\frac{1}{6}a^2\nabla^2\mathbf{u}\right)} +$$

Stokes drag

$$\boxed{-\frac{1}{2}\rho_f \frac{d}{dt}\left(\frac{d\mathbf{x}}{dt}-\mathbf{u}-\frac{1}{10}a^2\nabla^2\mathbf{u}\right) +}$$

Added mass

$$\boxed{-\frac{9\mu}{2a}\int_0^t \frac{d\tau}{\sqrt{\pi\nu(t-\tau)}}\frac{d}{d\tau}\left(\frac{d\mathbf{x}}{d\tau}-\mathbf{u}-\frac{1}{6}a^2\nabla^2\mathbf{u}\right)}$$

Basset « history » term

μ: viscosity; a: particle radius; ρ_f: fluid density; ρ_p: particle density

(from Basset-Boussinesq-Oseen, « BBO equation »)

Simplified equation

$$\frac{d^2\mathbf{x}}{dt^2} = \delta \cancel{\frac{D\mathbf{u}}{Dt}} - \frac{1}{\tau_p}\left(\frac{d\mathbf{x}}{dt} - \mathbf{u}\right), \qquad \delta = \frac{\rho_f}{\rho_p}$$

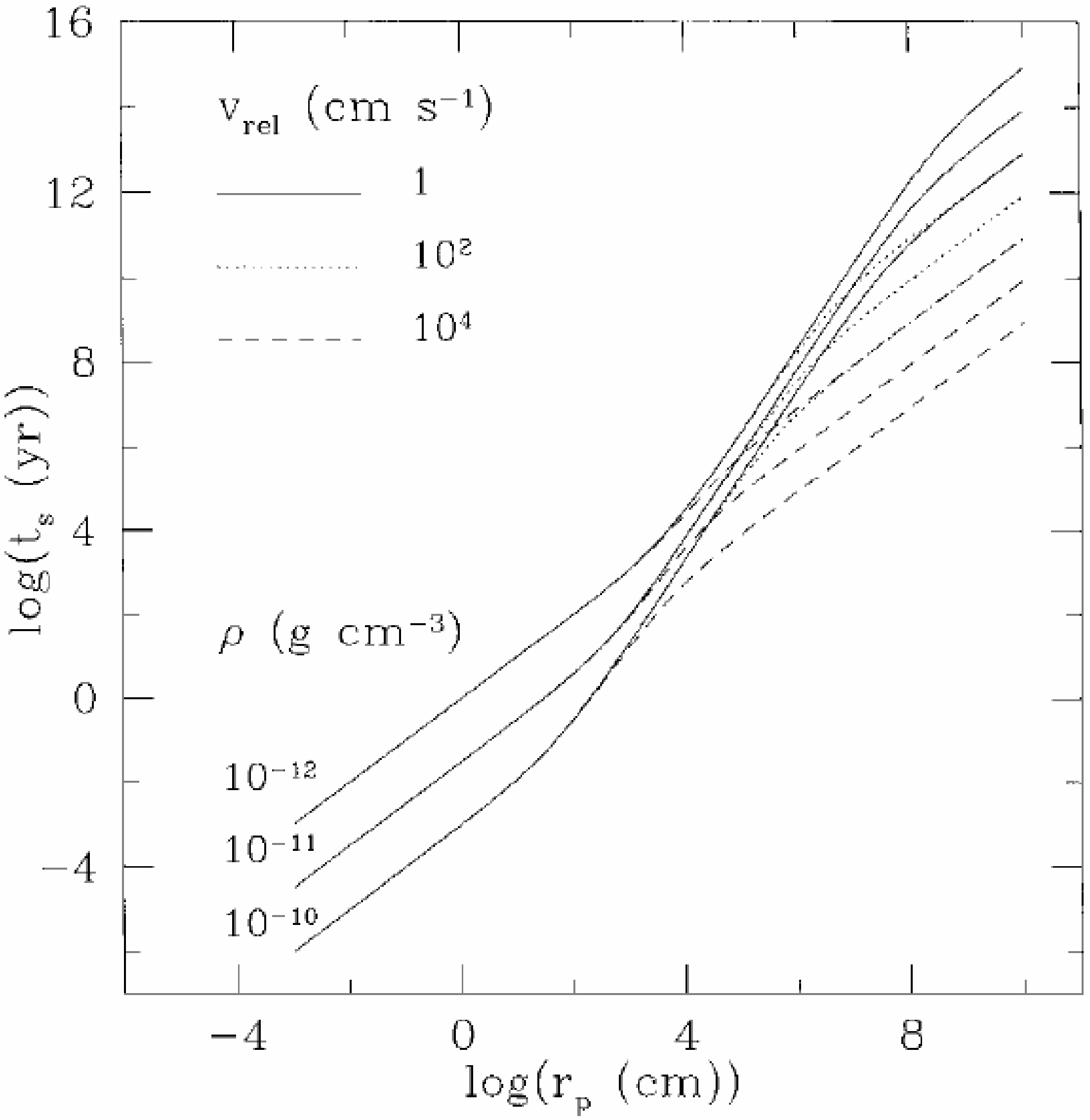
Stopping time :

$$\mathbf{v} = \frac{d\mathbf{x}}{dt} - \mathbf{u} \qquad \boxed{\tau_p = \frac{m|\mathbf{v}|}{F_g}}$$

$$F_g = \begin{cases} \dfrac{4\pi}{3} a^2 \rho_f v_{th} |\mathbf{v}| & a \ll \ell \quad \text{Epstein} \\ \\ \dfrac{C_D}{2} \pi a^2 \rho_f v_{th} |\mathbf{v}|^2 \dfrac{\mathbf{v}}{|\mathbf{v}|} & a \gg \ell \quad \text{Stokes} \end{cases}$$

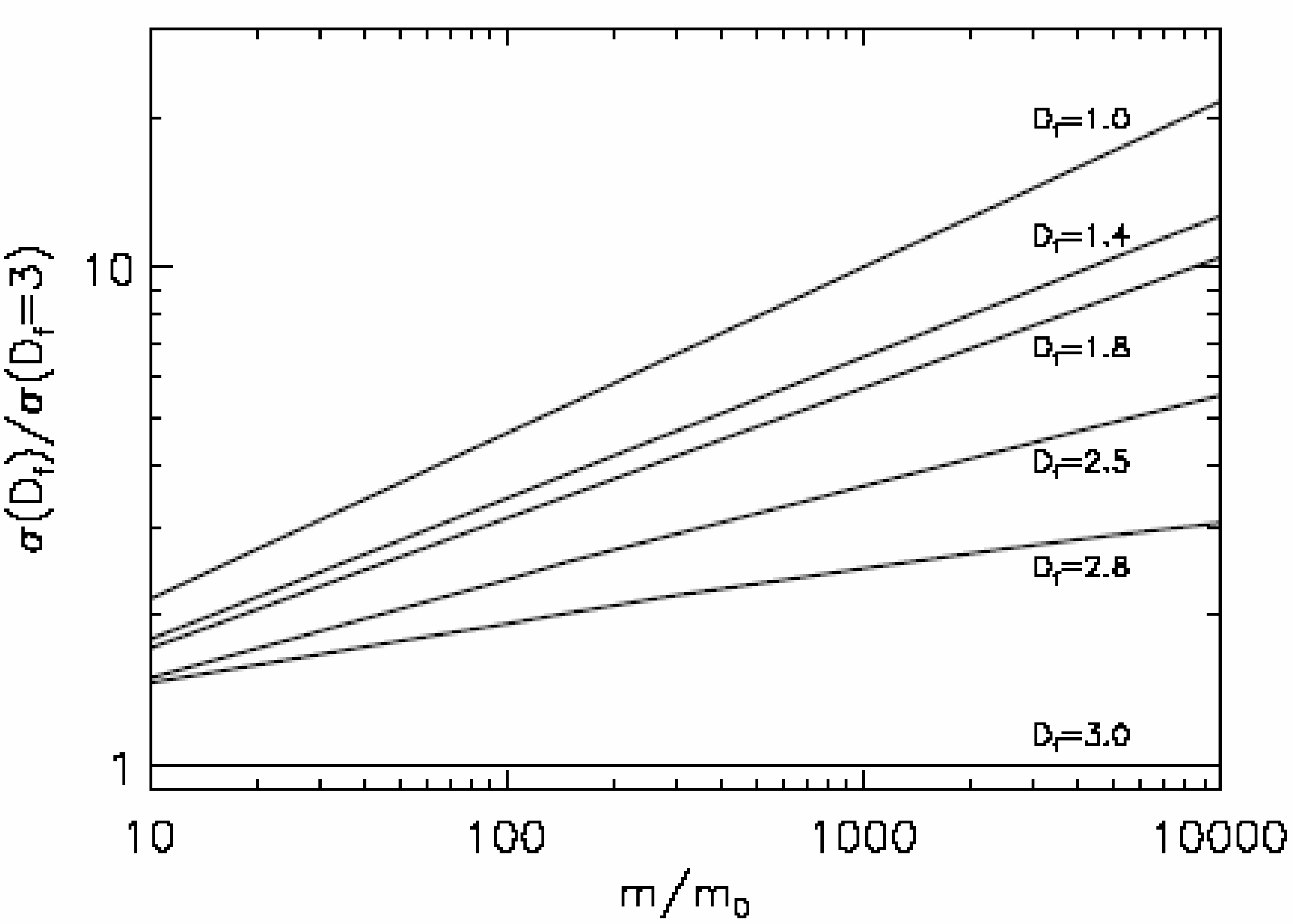
(Supulver, Lin 2000)

$\ell = (n \pi a_H^2)^{-1} \sim 5 \times 10^{-9}/\rho\ (\mathrm{g\ cm^{-3}}) \sim 1\text{-}10$ m



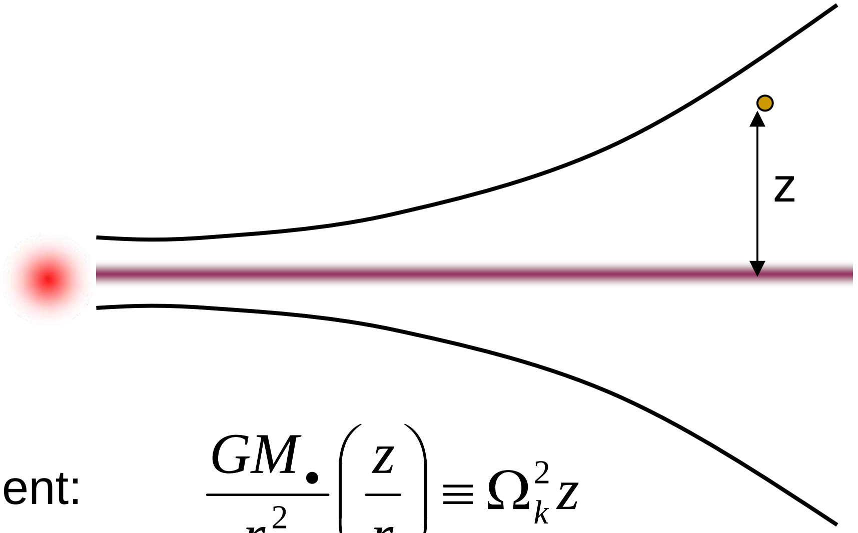
16
12
8
4
0
−4
−4
0
4
8
log(t_s (yr))
log(r_p (cm))
v_rel (cm s^-1)
1
10^2
10^4
ρ (g cm^-3)
10^-12
10^-11
10^-10

Gas friction for fractal particles



normalized projected areas of aggregates

Vertical settling



Central potential: vertical component:

$$\frac{GM_\bullet}{r^2}\left(\frac{z}{r}\right) \equiv \Omega_k^2 z$$

At the terminal speed:

$$\Omega_k^2 z = F_g \equiv \frac{4\pi}{3} a^2 \rho_f v_{th} v_z \longleftarrow F_g = \begin{cases} \frac{4\pi}{3} a^2 \rho_f v_{th} |\mathbf{v}| \\ \\ \frac{C_D}{2} \pi a^2 \rho_f v_{th} |\mathbf{v}|^2 \frac{\mathbf{v}}{|\mathbf{v}|} \end{cases}$$

$$\Longrightarrow \boxed{\tau_{settle} = \frac{z}{v_z} \approx \frac{10^3 (yr)}{a\ (cm)}}$$

Sub-keplerian rotation

Tangential gas velocity in the disk:

$$-\frac{v_\theta^2}{r} = -\frac{1}{\rho}\frac{dP}{dr} - \frac{GM_\bullet}{r^2}$$

$$-\frac{v_k^2}{r^2}$$

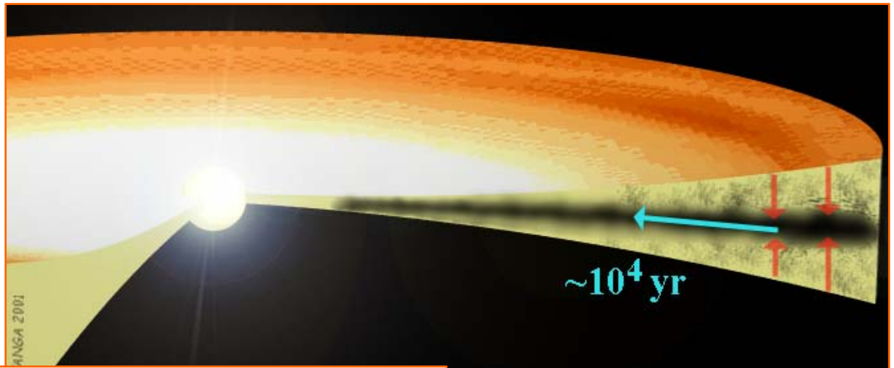
$$-\frac{v_k^2}{r}\left[1-\left(1-\frac{r\eta}{v_k^2}\right)^2\right] = -\frac{1}{\rho}\frac{dP}{dr}$$

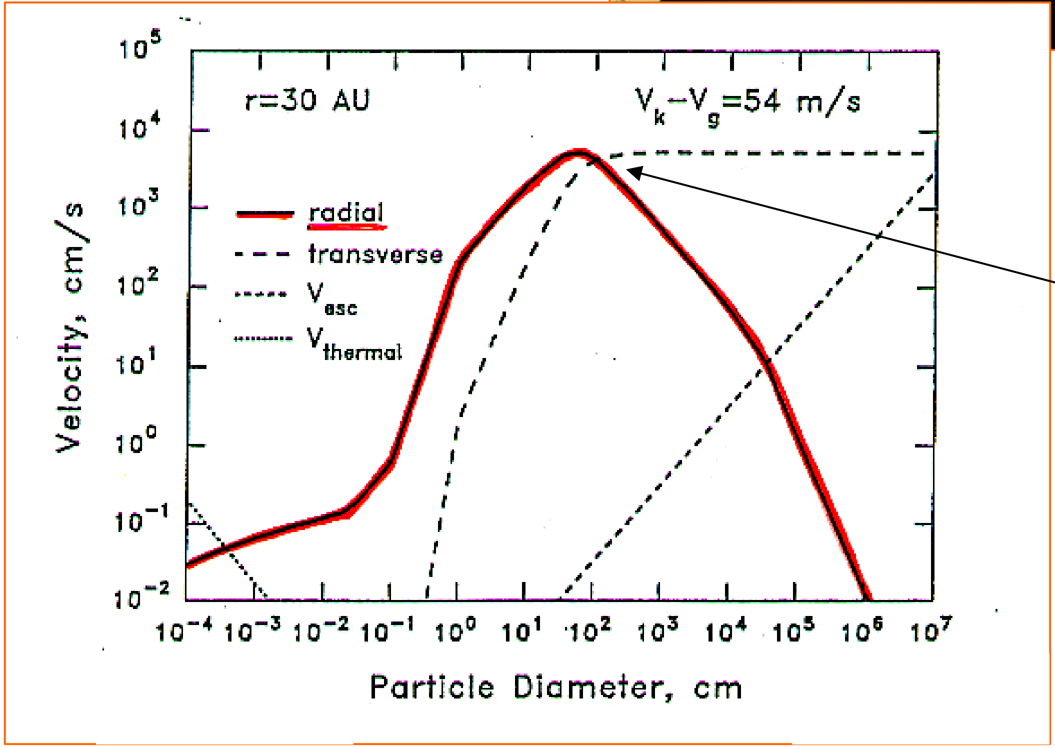
$$\eta \equiv \Omega_k (v_k - v_\theta)$$

$$\Longrightarrow \boxed{\eta \approx -\frac{1}{2\rho}\frac{dP}{dr} \approx 10^{-3}}$$

Sunward dust fall

Dust particles run headwind





$St \sim \tau_p \Omega \sim 1$

Weidenschilling 1977-..

Classic « Old » Planet Formation Scenario

Small density fluctuations could become unstable and form large planetesimals.

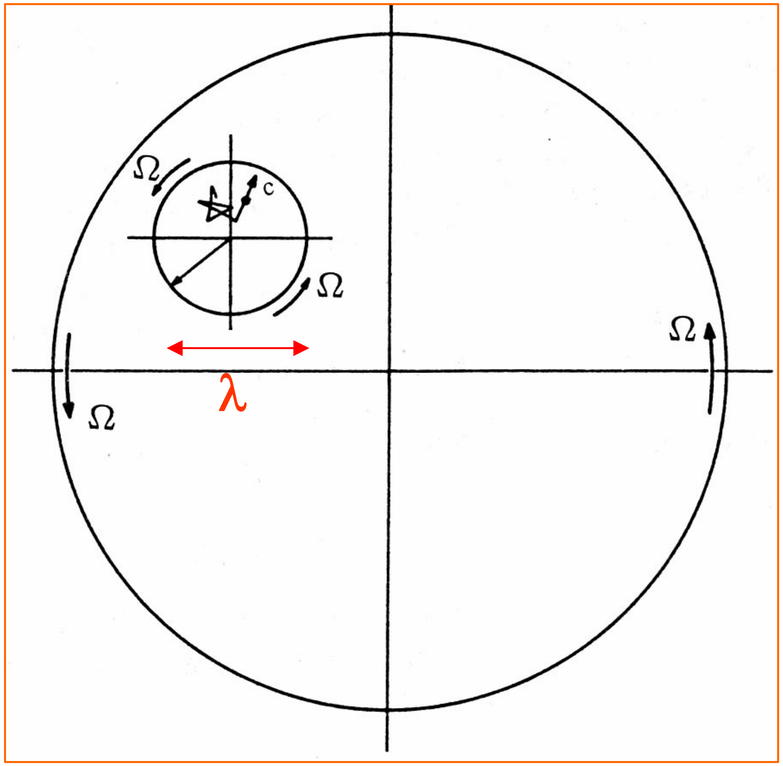
From linear perturbations analysis:

$$F(\lambda) = 4\pi^2 c^2 - 4\pi G \Sigma \lambda + \lambda^2 \Omega^2$$

Σ_p = surf. density of solids;
c = velocity dispersion

Safronov 1969
Goldreich, Ward 1973

It requires:
- settling
- fast collisional dissipation



Unstable wavelengths

$$F(\lambda) = 4\pi^2 c^2 - 4\pi G\Sigma\lambda + \lambda^2\Omega^2$$

$$\lambda^* = \frac{2\pi^2 G\Sigma}{\Omega^2} \qquad c^* = \frac{\pi G\Sigma}{\Omega}$$

At 5 AU, in a "minimum mass" nebula, λ_c = 5.5 10^{-4} AU

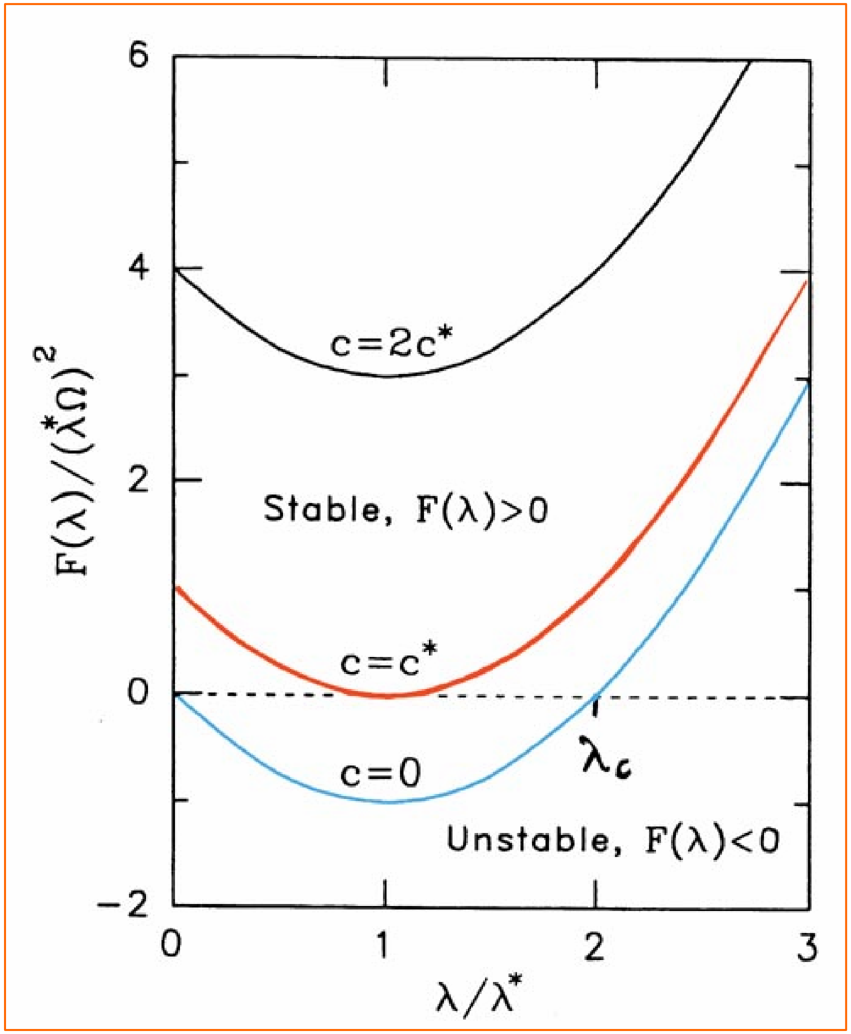
$\Sigma \sim r^{-3/2}$

$\lambda_c \sim r^{3/2}$

$c^* \sim$ const.

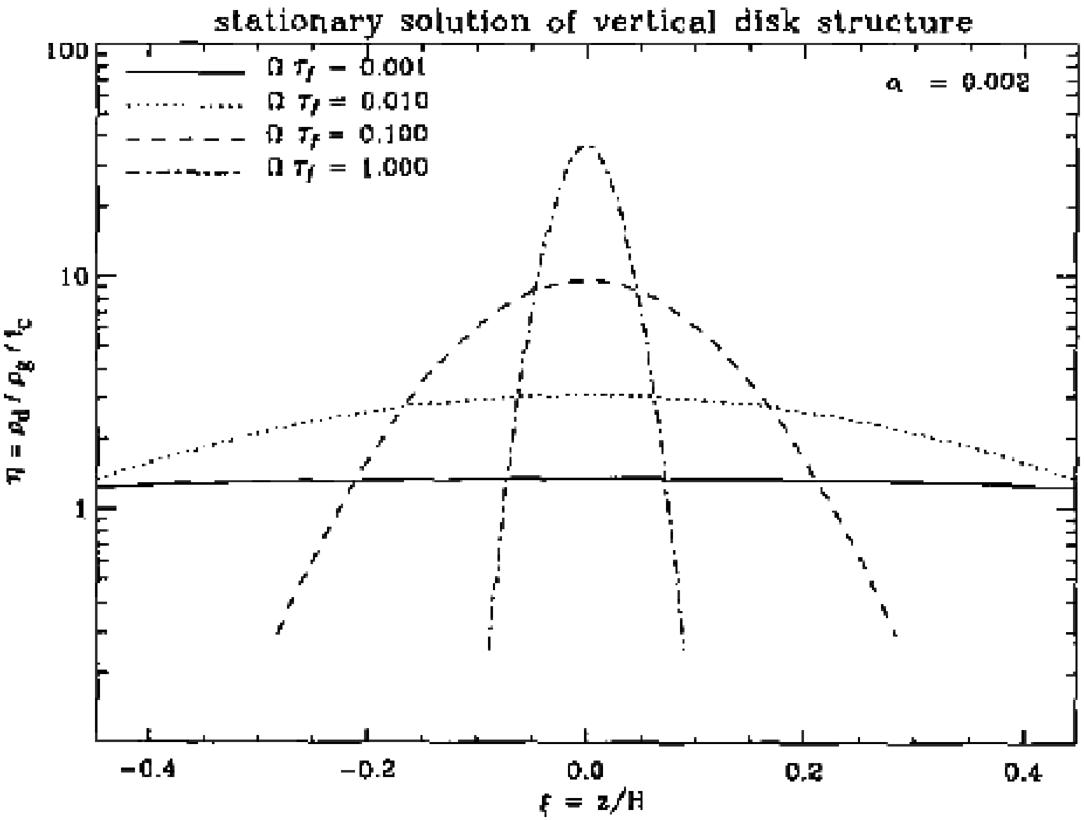
Equivalent to the Toomre crit.:

$$Q_p = \frac{\Omega c^*}{\pi G\Sigma} < 1$$



Turbulent diffusivity

Global disk turbulence is very efficient in preventing settling



Dubrulle et al. 1995

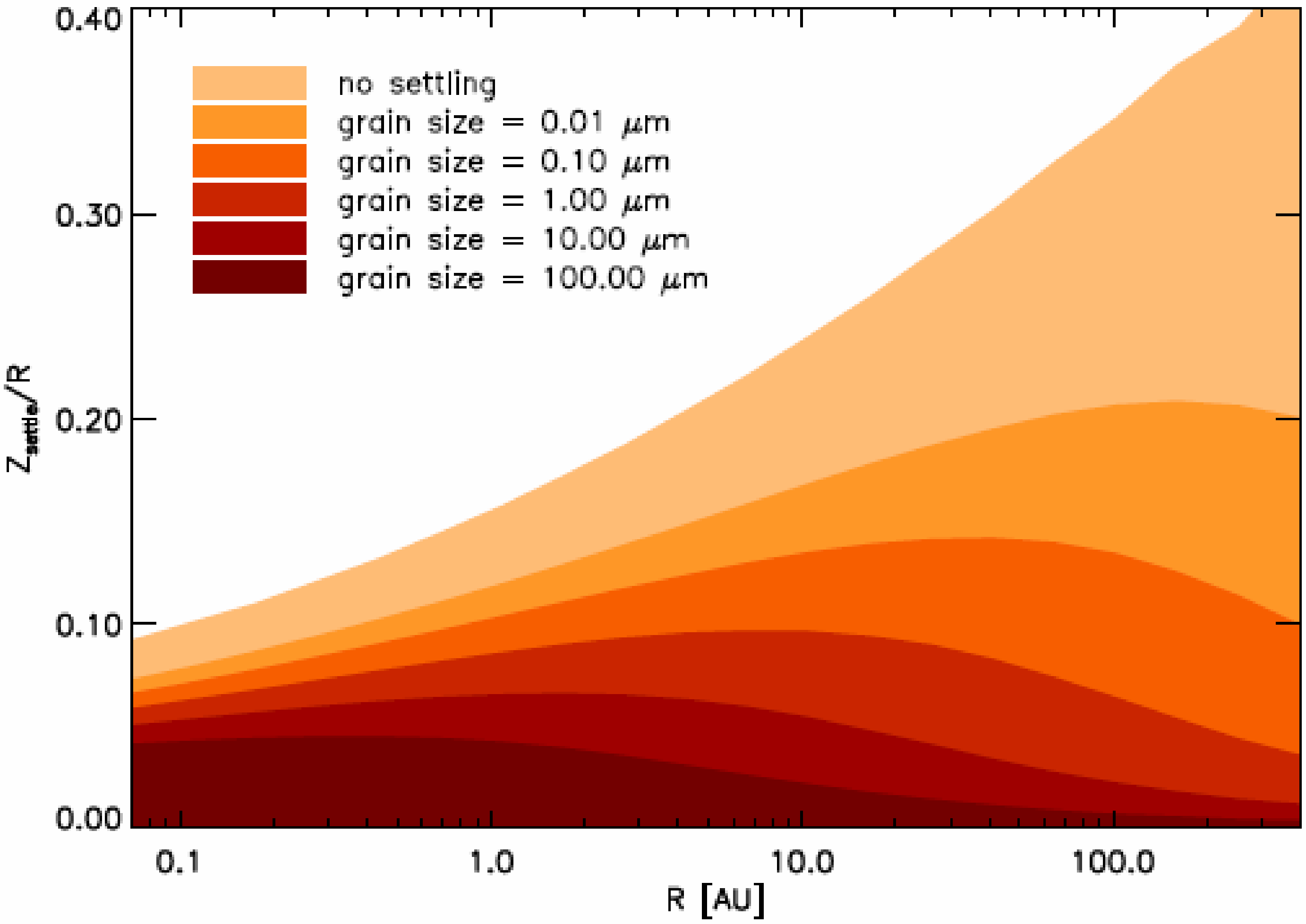
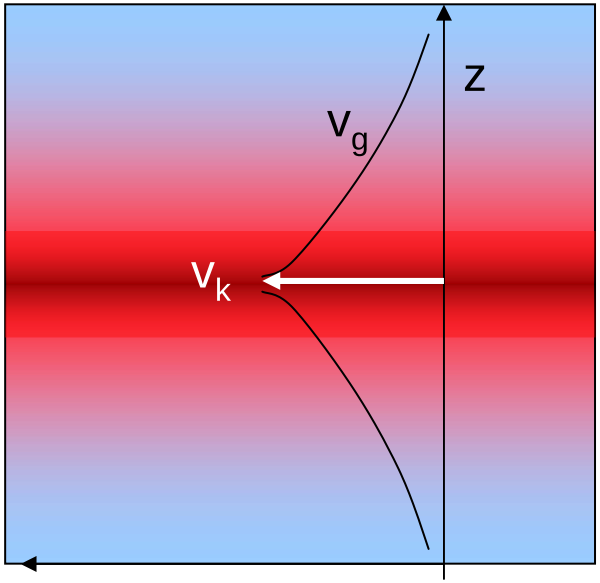
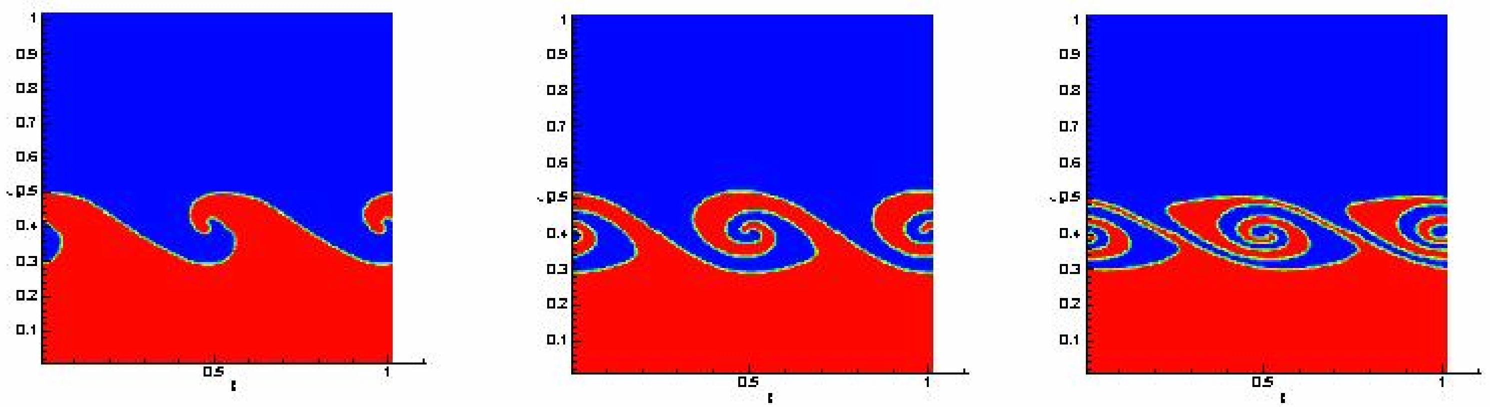


Fig. 1.4. Sedimentation of small grains: the figure shows how deep grains of a certain size sediment into the disk after equilibrium between sedimentation and turbulent mixing has set in for a turbulent α-parameter of $\alpha = 10^{-4}$ (Dullemond and Dominik, 2004).

Kelvin-Helmoltz instability in a « laminar » nebula



- Settled dust create an overdense layer:
  - back reaction on gas
  - vertical velocity gradient (shear)
- Kelvin-Helmoltz instability?

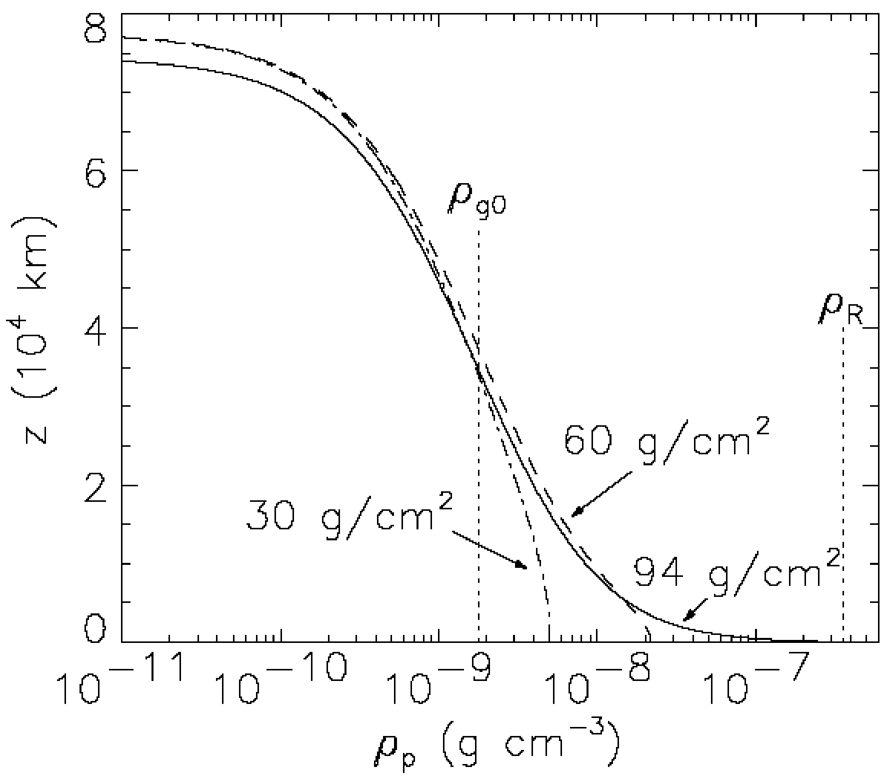


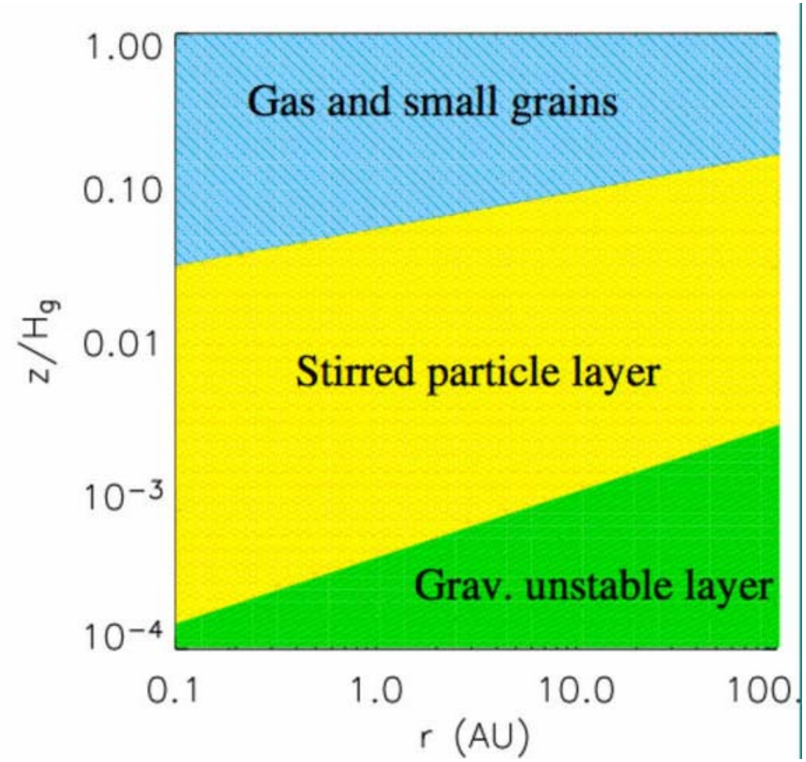
Could K-H turbulence prevent the gravitational instability of the dust layer?

- Requirements:
  - Critical mass density ~10^{-7} g cm^{-3} (10^{3} x gas density)
  - → thickness $h<10^{-5}$ H ($a<10^{-8}$-10^{-10} ??)
  - « Perfect » dissipation by collisions
  - Gas pressure support for particles $\tau_s<(G\rho_p)^{-1/2}$
  - → even higher densities required (x 10^{4})

Weak turbulence will prevent direct collapse

A new perspective in an enriched nebula

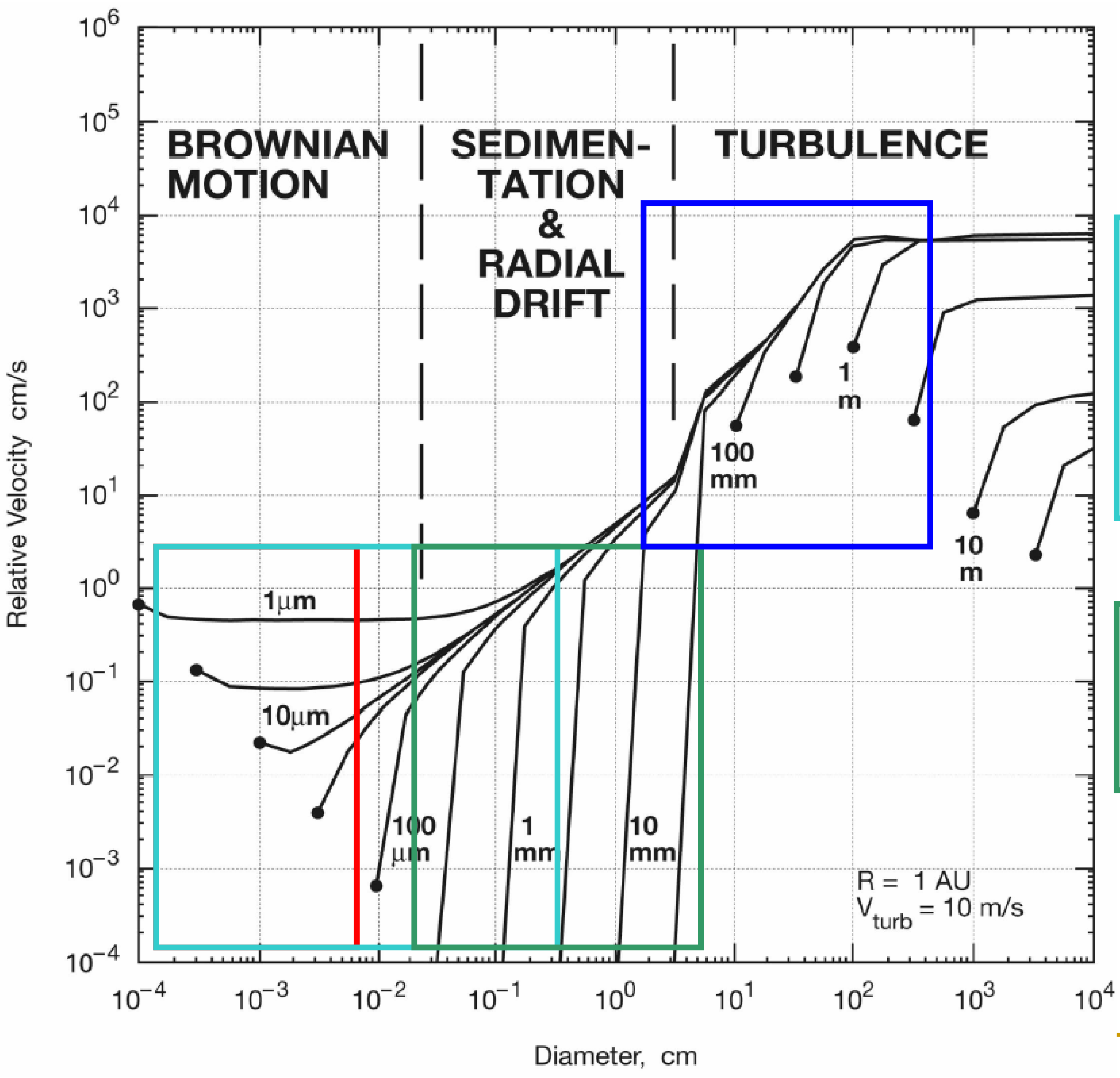




The disk could be « K-H stable » if stratification increases

Youdin & Shu 2002

Putting dust in a disk…



$D_f<2$, τ_s~single particle

Decoupling from small-scale gas motion

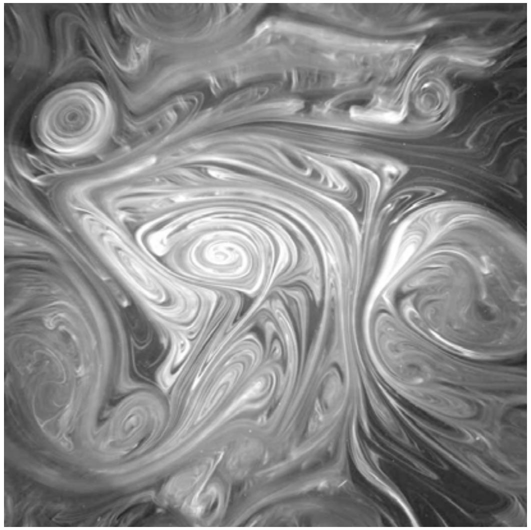
Interaction with smaller particles: $D_f \rightarrow 3$

Large grains sediment faster: sweeping of small grains

Erosion Turbulence ?

The danger of simplicity

- Flow topology, inertial particles
- Density and velocity are correlated…
- The simple « homogeneous » approach can be misleading
- « Turbulent », not « random »



What is fluid turbulence?

« Turbulence » is the contemporary presence of different scales of motion, interacting by energy transfer.

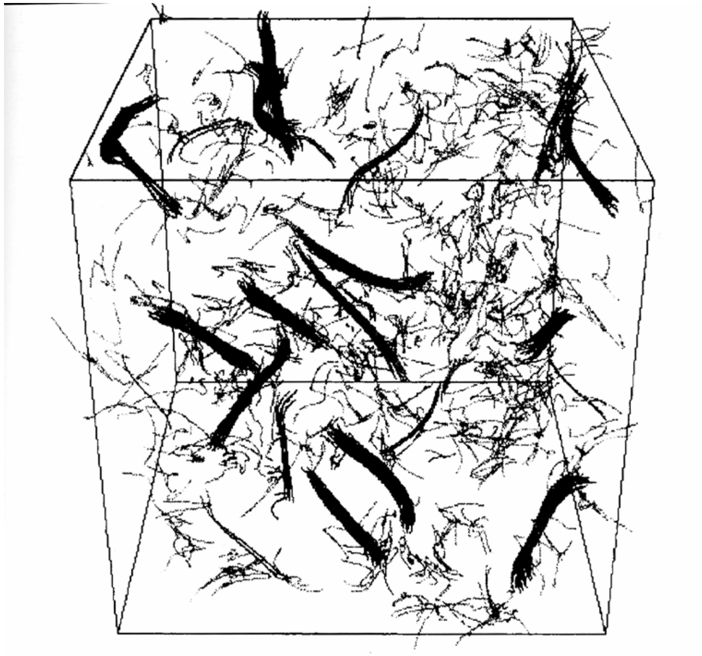


Fig. 7.4. Intermittent vortex filaments in a three-dimensional turbulent fluid simulated on a computer (She, Jackson and Orszag 1991).

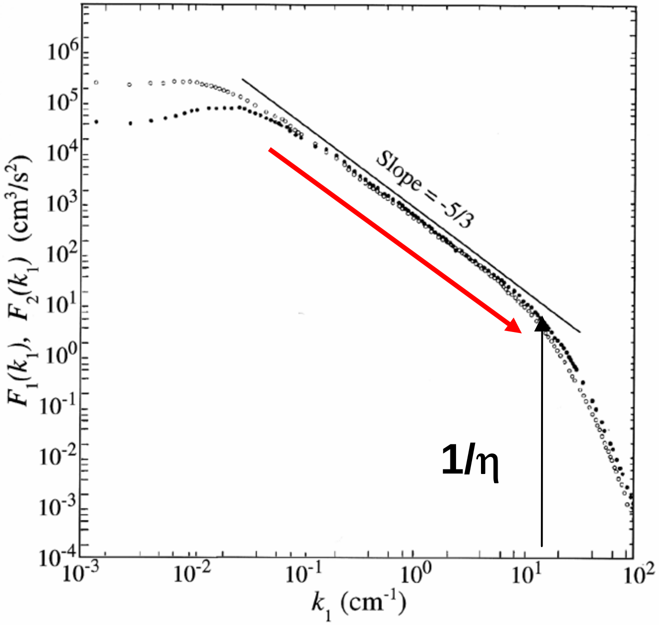
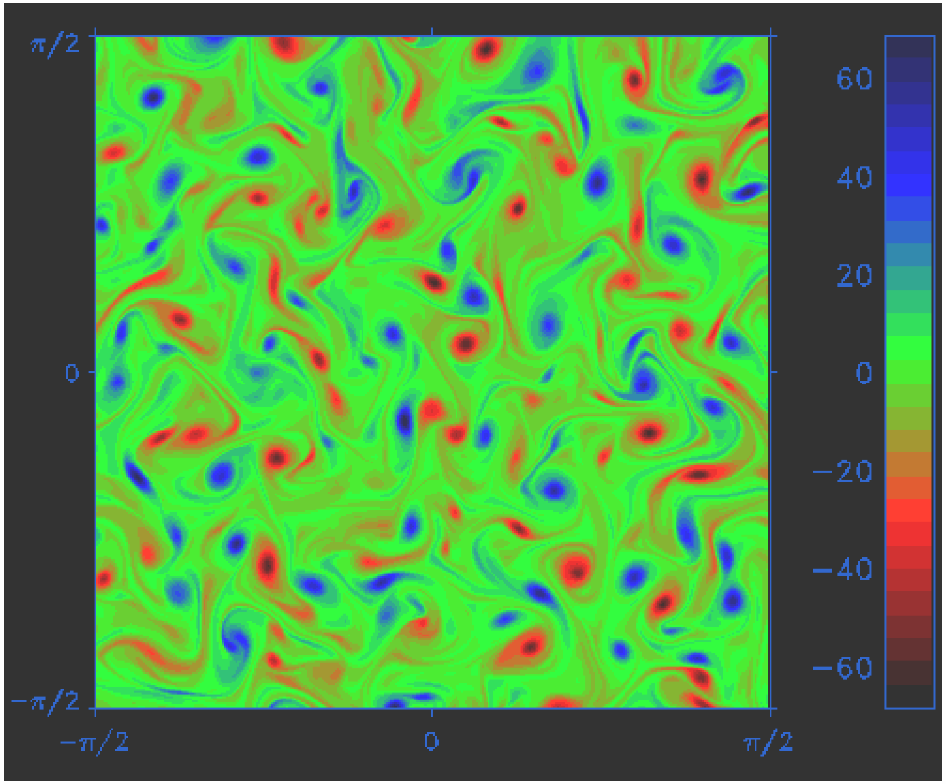
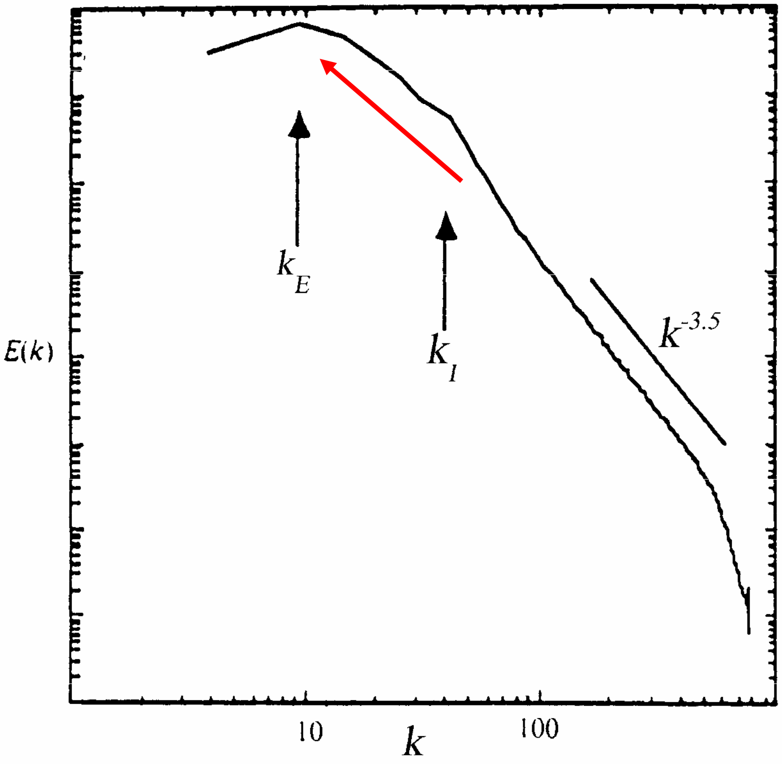


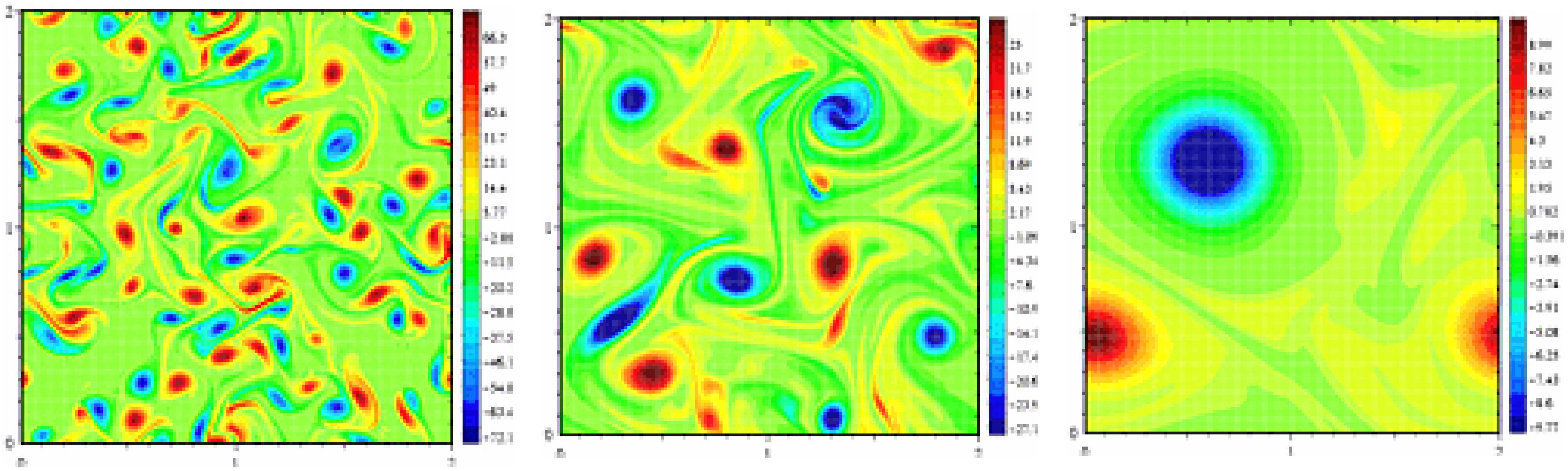
Fig. 5.7. log–log plot of the energy spectra of the streamwise component (white circles) and lateral component (black circles) of the velocity fluctuations in the time domain in a jet with $R_\lambda = 626$ (Champagne 1978).

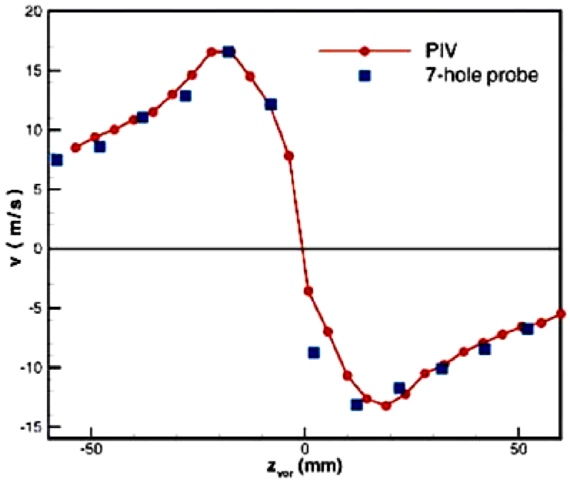
Turbulence in 2D





2D decaying turbulence

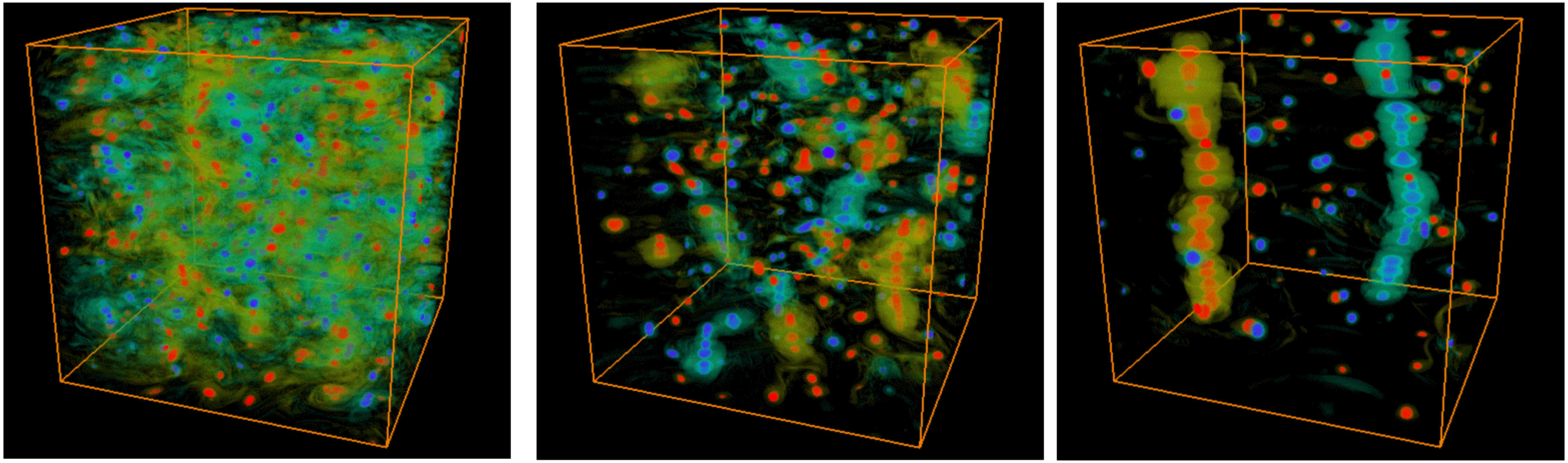




Low energy transfer inside vortices:
small scales are ABSENT

Low relative velocities

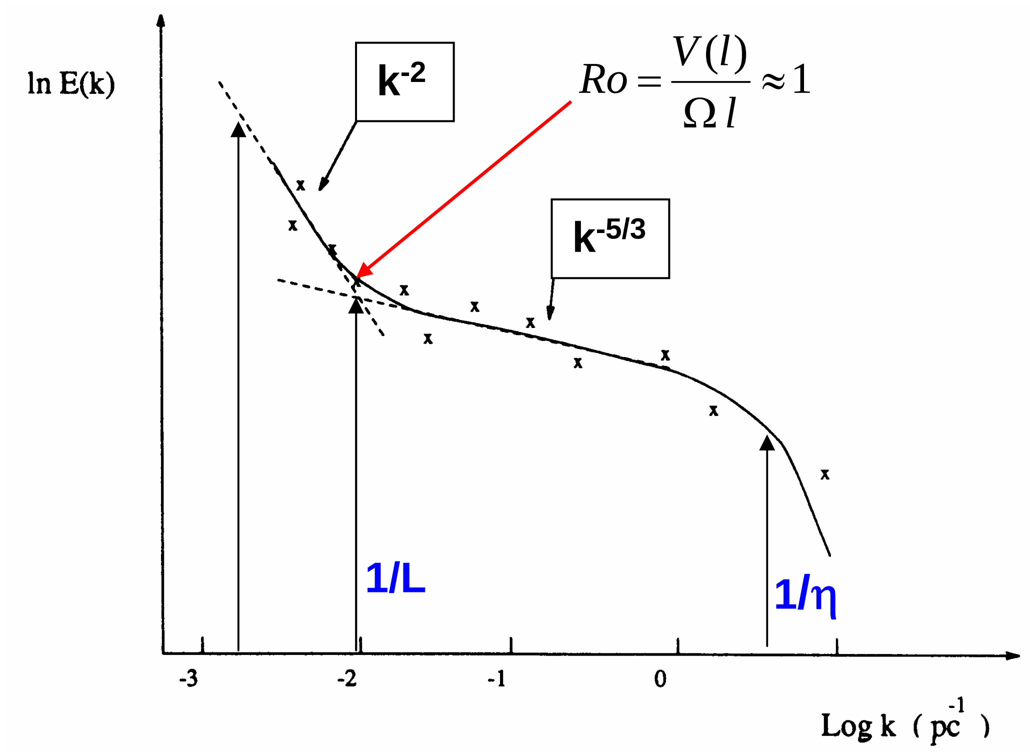
From 3D to 2D turbulence



McWilliams, 1994

Rossby number $$Ro = \frac{V(l)}{\Omega\, l} \approx 1$$

Tubulence scales, speeds, structures, rotation…



$$Ro = \frac{V(l)}{\Omega\, l} \approx 1$$

Energy spectrum of the galactic disk
(Vereshchagin, Solov’ev 1990)

$$\mathrm{Re} \approx \frac{\alpha C_s H}{\nu_m} \approx \left(\frac{L}{\eta}\right)^{\frac{4}{3}}$$

$$\omega(l) \approx \frac{V(l)}{l} \approx \Omega \left(\frac{l}{L}\right)^{-\frac{2}{3}}$$

$$L \approx H\sqrt{\alpha}\left(\frac{\Omega}{\omega(L)}\right)^{\frac{1}{2}}$$

Are disks turbulent?

- « Observed » accretion rates: ~10-8 Mo/yr
- Molecular viscosity if by far too weak to sustain the observed accretion
- Re $= 3\times 10^{13}\left(\frac{M_*}{M_\odot}\right)^{-1/2}\left(\frac{\bar{n}}{7\times 10^{14}\ \mathrm{cm}^{-3}}\right)$

$$\times\left(\frac{\bar{T}}{930\ \mathrm{K}}\right)^{1/2}\left(\frac{r_{\mathrm{in}}}{10^{11}\ \mathrm{cm}}\right)^{-1}\left(\frac{r_{\mathrm{o}}}{10^{3}\ \mathrm{AU}}\right)^{-1/2}$$

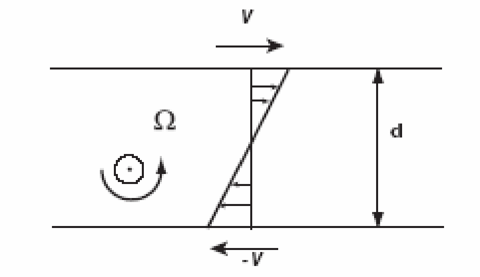
(Longaretti 2003; Dubrulle et al. 2004)

Main (in)stability issues

- A keplerian profile is linearly stable wrt axisym. perturbations (Rayleight crit.):

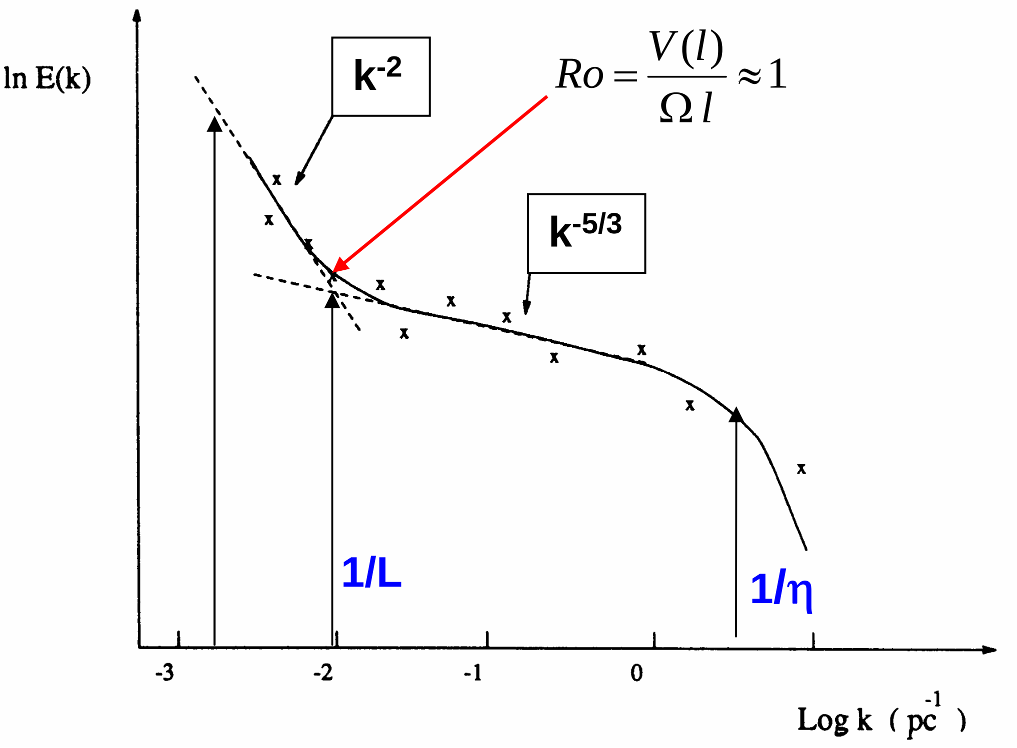
$$\frac{\mathrm{d}\left(r^2\Omega\right)^2}{\mathrm{d}r} > 0$$

- Finite amplitude disturbances (nonlinear)
  Taylor Couette experiment
  (Dubrulle 1993, Richard and Zhan 1999,
  Lesur & Longaretti 2005)

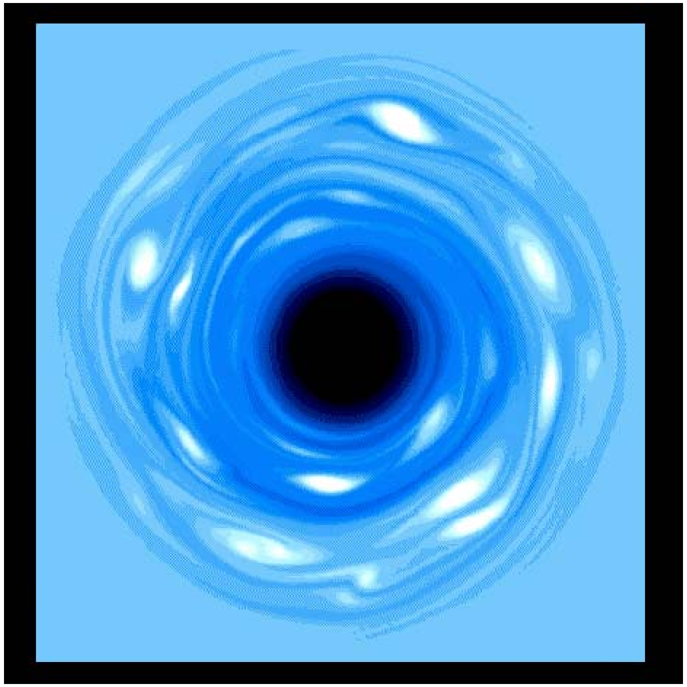


- Vertical magnetic field (Balbus, Hawley 1991), « arbitrarly weak »
  → Magneto Rotational Instability

Large scale structures in disks?



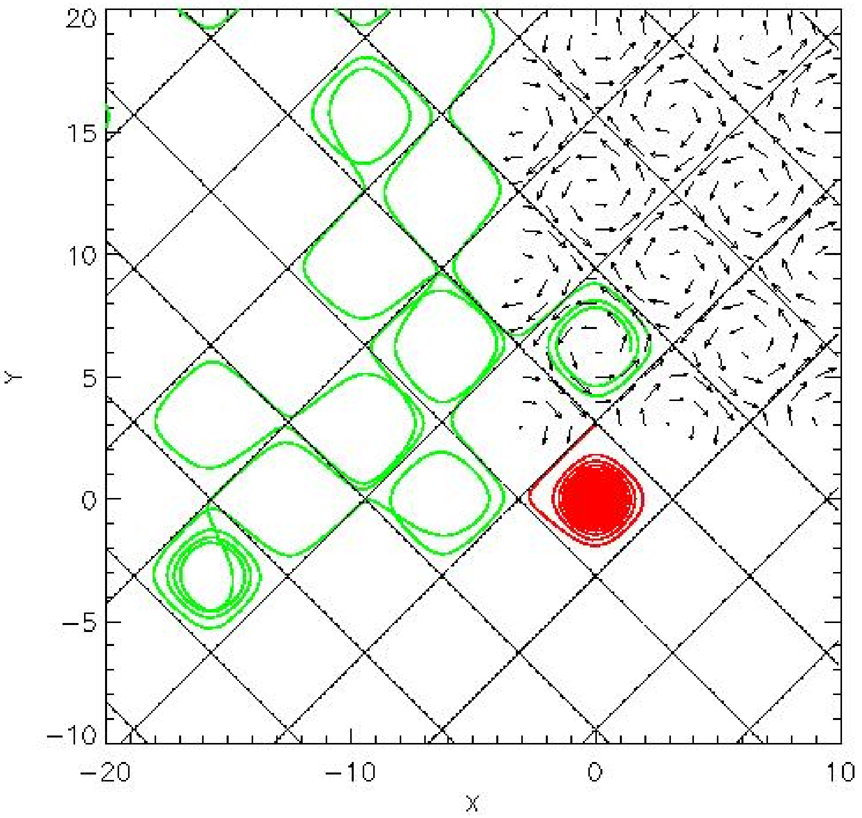
Energy spectrum of the galactic disk
(Vereshchagin, Solov'ev 1990)



Bracco et al 1998
Anticyclons can form and survive
in a keplerian shear

Effects of particle inertia: simple flows

$$\frac{d^2\mathbf{x}}{dt^2} = \delta \frac{D\mathbf{u}}{Dt} - \frac{1}{\tau_p}\left(\frac{d\mathbf{x}}{dt} - \mathbf{u}\right)$$



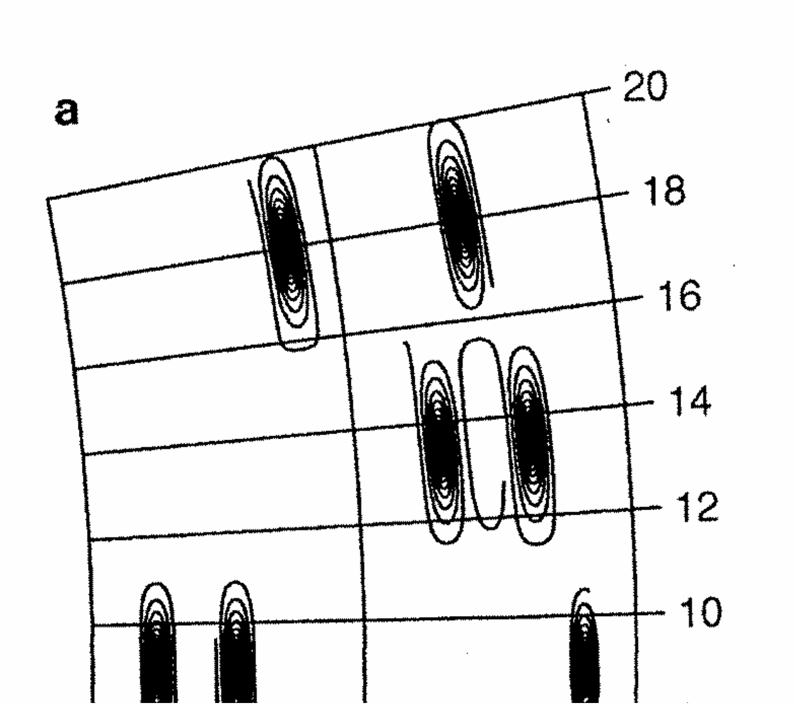
δ>1

δ<1

Chaotic diffusion of particles in simple, stationary flows

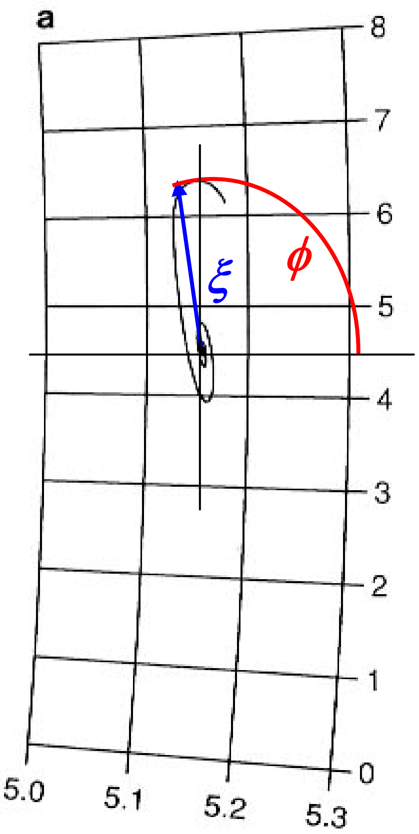
Effects of particle inertia: adding global rotation

$$\boxed{\frac{d^2\mathbf{r}}{dt^2} = \delta\frac{D\mathbf{u}}{Dt} - \frac{1}{\tau_f}\left(\frac{d\mathbf{r}}{dt} - \mathbf{u}\right)}\boxed{-2\mathbf{\Omega}\times\left(\frac{d\mathbf{r}}{dt} - \delta\mathbf{u}\right) + \left(\Omega^2 r - \frac{GM_*}{r^2}\right)(1-\delta)\hat{\mathbf{r}}}$$



$\Omega \neq 0$, $\delta \rightarrow 0$,
$Ro = U/2\Omega L < 1$

Anticyclonic vortices capture small planetesimals



$$\frac{d^2\xi}{dt^2} = \boxed{\frac{1}{\xi}\Bigg[\left(\xi\frac{d\phi}{dt}\right)^2} - \delta v_\phi^2\Bigg] + \boxed{+2\Omega\left(\xi\frac{d\phi}{dt} - \delta v_\phi\right)} - t_e^{-1}\left(\frac{d\xi}{dt} - v_\xi\right)$$

Tanga, Babiano, Dubrulle, Provenzale 1996

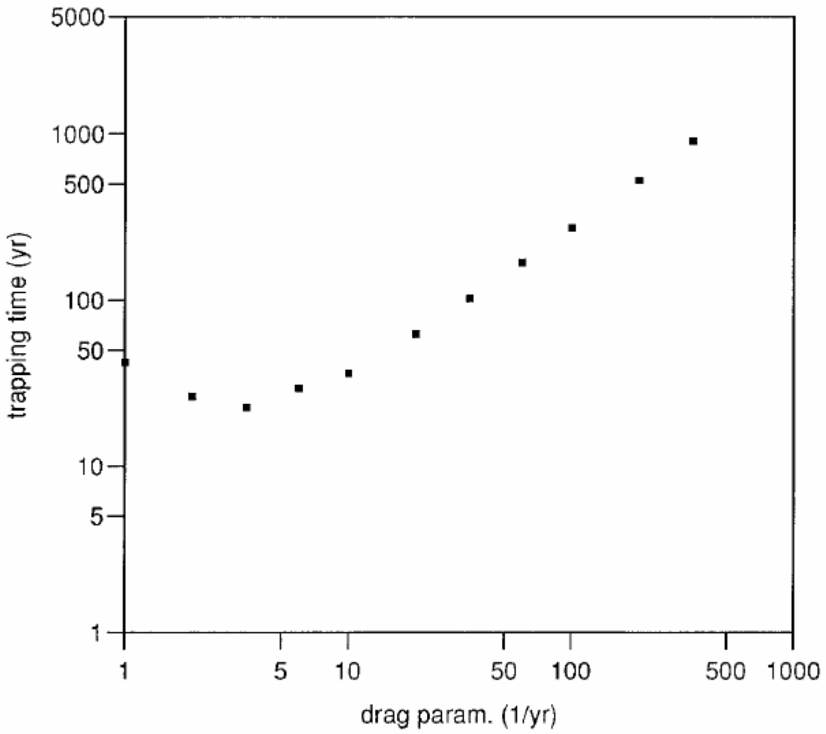


FIG. 5. Time taken by a dust particle to reach the center of an anticyclonic vortex at 5 AU as a function of the value of the drag parameter τ_f^{-1}.

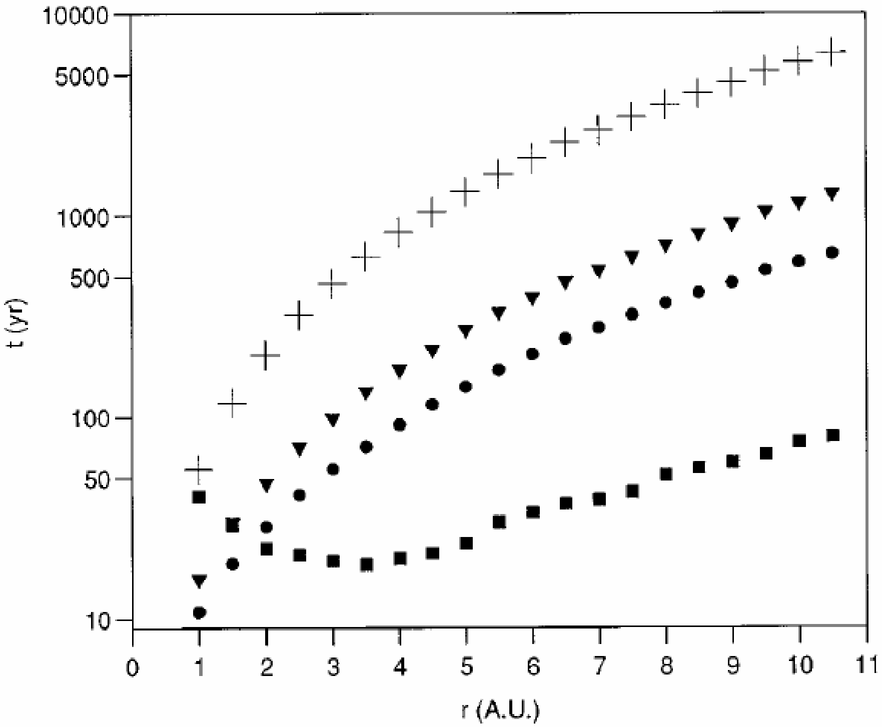
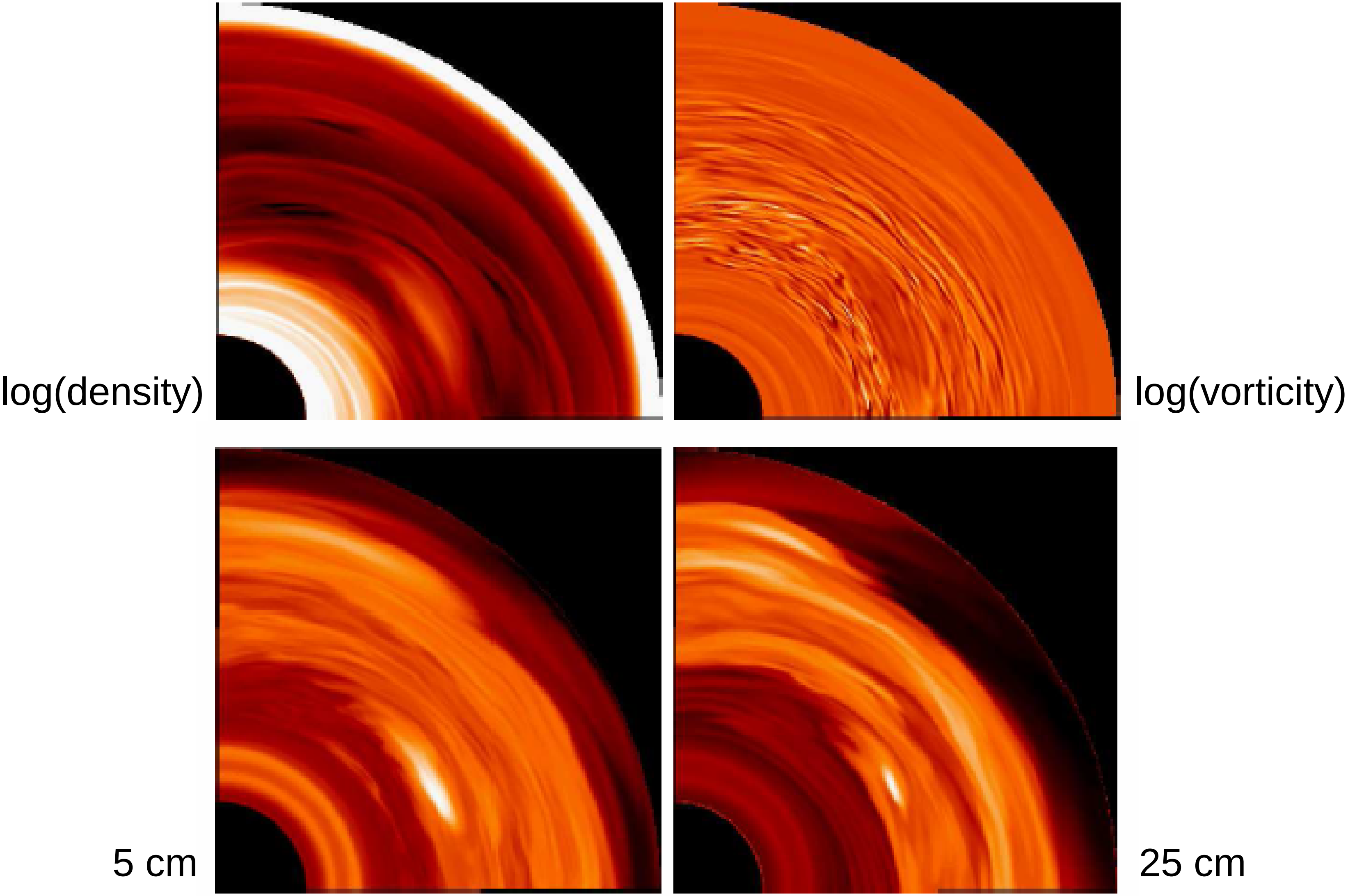


FIG. 7. Time taken by a dust particle to reach the center of an anticyclonic vortex as a function of the distance from the Sun. Squares refer to $\tau_f^{-1} = 5$ year^{-1}, circles to $\tau_f^{-1} = 50$ year^{-1}, triangles to $\tau_f^{-1} = 100$ year^{-1}, and crosses to $\tau_f^{-1} = 500$ year^{-1}.



Fromang, Nelson, MNRAS, submitted

Role of large-scales structures: summary

- Vortices can induce *low* relative velocities
- They efficiently affect particle distribution depending upon:
  - St~1
  - Lifetime
  - Displacement
- In-homogeneous growth →

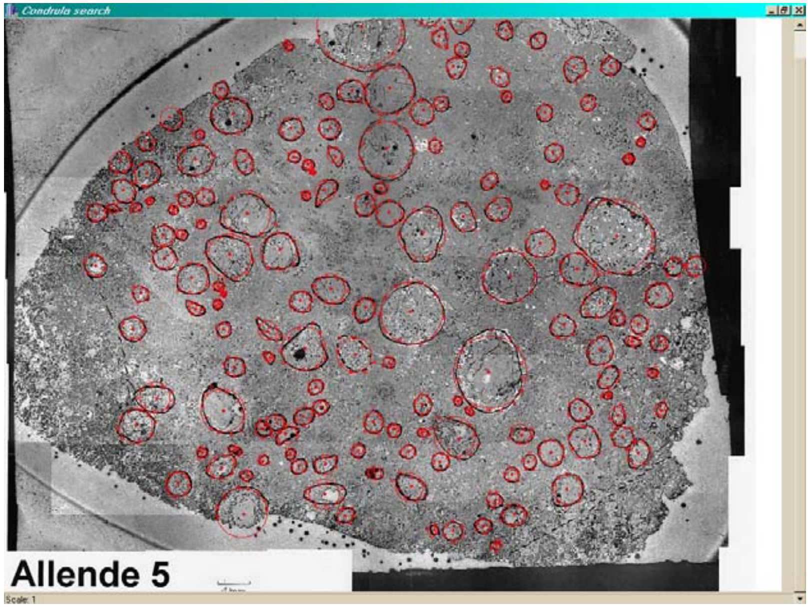
  role of gravity? Collision rates?

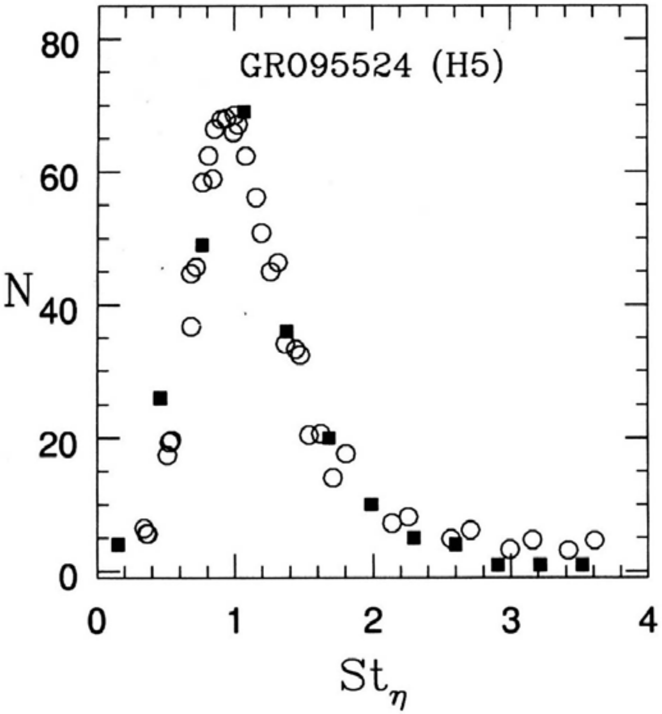
- Could they directly form « planets »?

(Klahr Bodenheimer 2006)

Small scales

3D isotropic turbulence? (Ro >> 1)

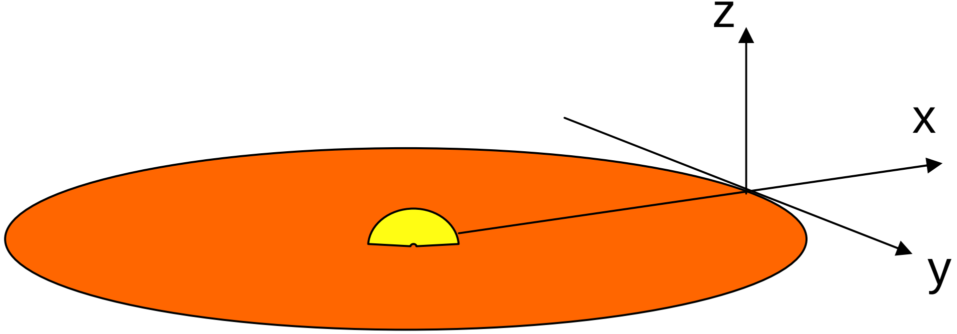




Cuzzi et al. 2001
3D turbulence (St~0.2 - 6)

An extreme example:
the paradox of purely « diffusive » turbulence

Johansen, Henning, Klahr
Dust sedimentation and self-sustained Kelvin Helmoltz turbulence in protoplanetary disks
A&A submitted



ε = local dust/gas mass ratio

$$\frac{\partial u_x}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})u_x = 2\Omega_0 u_y - \frac{1}{\gamma}c_{\mathrm{s}}\Omega_0\beta - \frac{\epsilon}{\tau_{\mathrm{f}}}\left(u_x - w_x\right) ,$$

$$\frac{\partial u_y}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})u_y = -\frac{1}{2}\Omega_0 u_x - \frac{1}{\rho}\frac{\partial P}{\partial y} - \frac{\epsilon}{\tau_{\mathrm{f}}}\left(u_y - w_y\right) ,$$

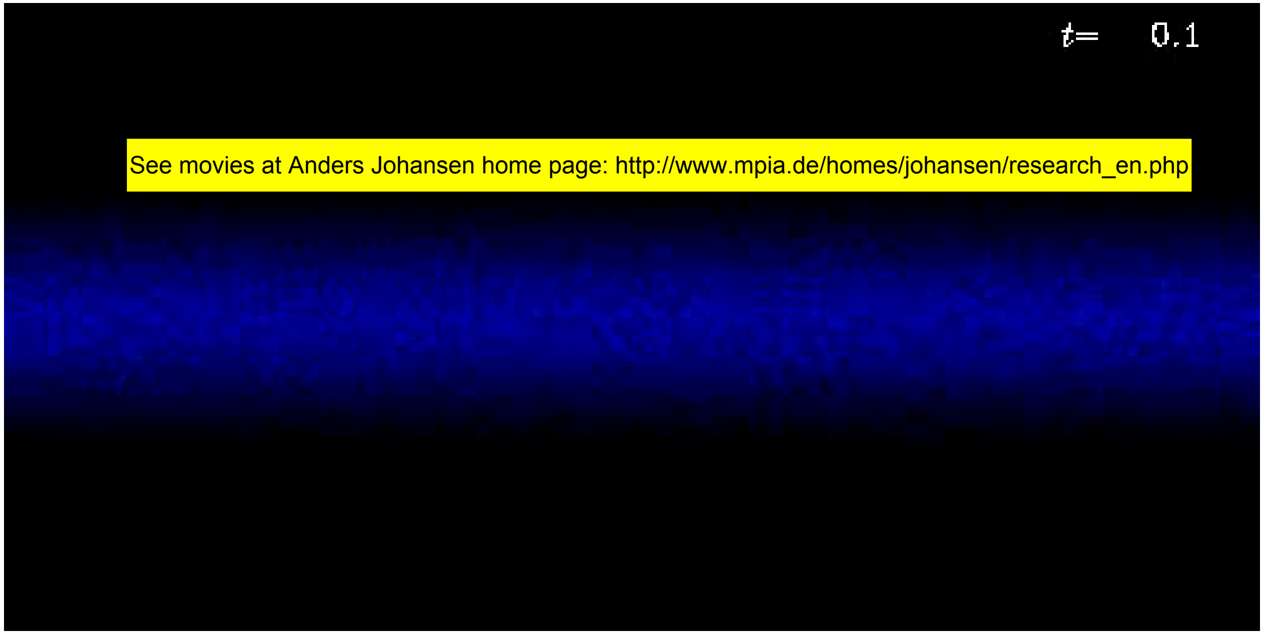
$$\frac{\partial u_z}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})u_z = -\Omega_0^2 z - \frac{1}{\rho}\frac{\partial P}{\partial z} - \frac{\epsilon}{\tau_{\mathrm{f}}}\left(u_z - w_z\right) .$$

$$\frac{\partial v_x^{(i)}}{\partial t} = 2\Omega_0 v_y^{(i)} - \frac{1}{\tau_{\mathrm{f}}}\left(v_x^{(i)} - u_x\right) ,$$

$$\frac{\partial v_y^{(i)}}{\partial t} = -\frac{1}{2}\Omega_0 v_x^{(i)} - \frac{1}{\tau_{\mathrm{f}}}\left(v_y^{(i)} - u_y\right) ,$$

$$\frac{\partial v_z^{(i)}}{\partial t} = -\Omega_0^2 z^{(i)} - \frac{1}{\tau_{\mathrm{f}}}\left(v_z^{(i)} - u_z\right) .$$

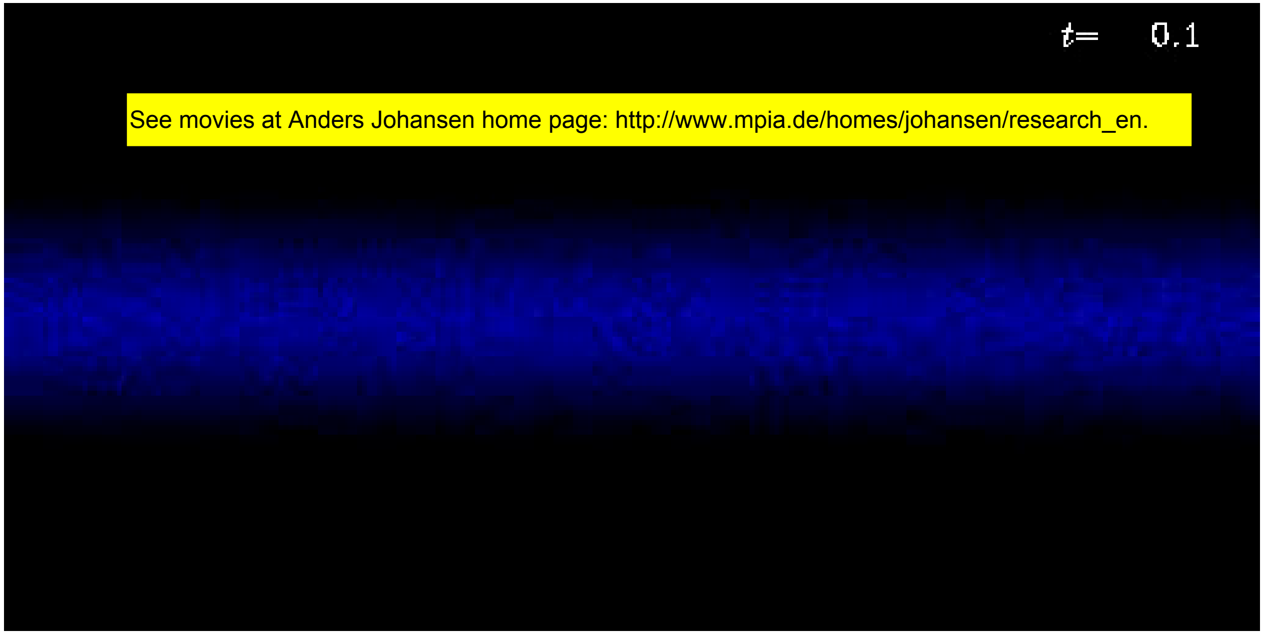
Initial conditions for gas: $\rho(z) = \rho_1 \mathrm{e}^{-z^2/(2H^2)}$



See movies at Anders Johansen home page: http://www.mpia.de/homes/johansen/research_en.php

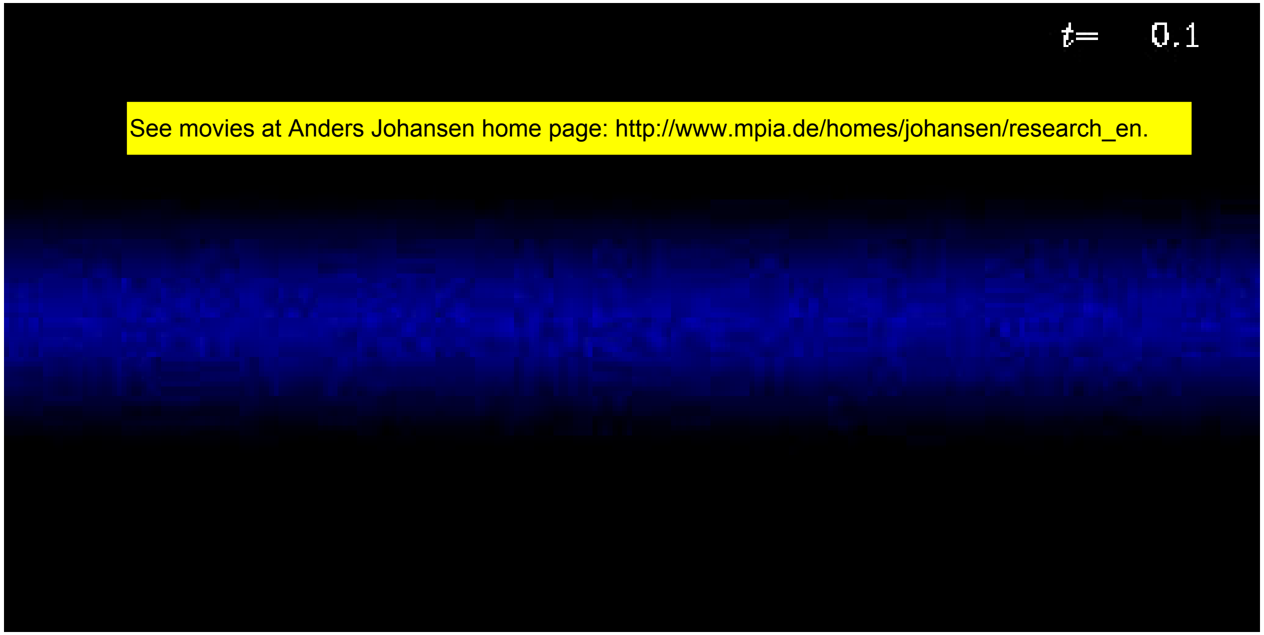
1 cm particles

10 cm particles



See movies at Anders Johansen home page: http://www.mpia.de/homes/johansen/research_en.

1 m particles



See movies at Anders Johansen home page: http://www.mpia.de/homes/johansen/research_en.

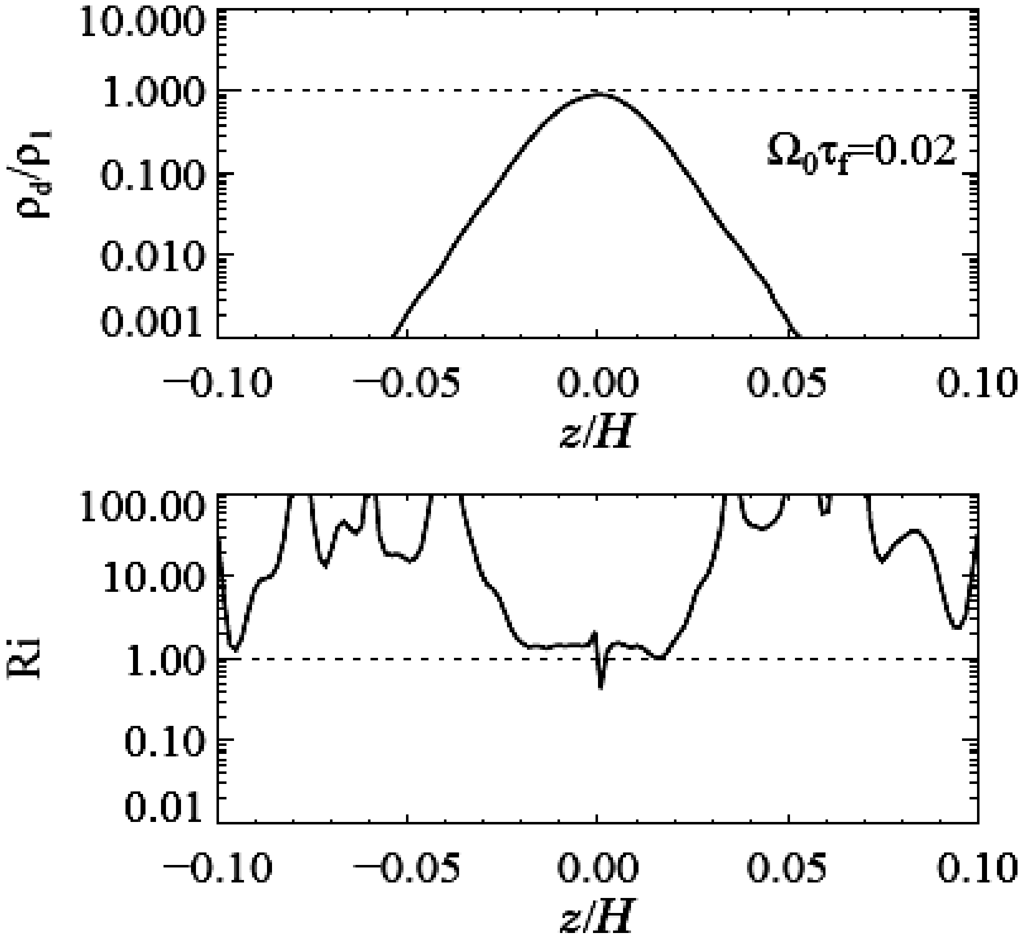


FIG. 4.— Dust density and Richardson number averaged over the azimuthal direction and over time. The dust-to-gas ratio in the mid-plane is close to unity and falls rapidly outwards. The Richardson number is approximately constant in the mid-plane and has a value around unity.

$$\mathrm{Ri} = \frac{g_z \partial \ln \rho / \partial z}{(\partial u_y / \partial z)^2} \, .$$

Conclusions

- Turbulence could be unavoidable
  - as byproduct of particle settling in a laminar nebula…
  - as global turbulence
- Particle distribution in position and velocity cannot be disentangled
- → inhomogeneous growth could be « the rule » in many situations
  - for building chondrulae
  - for the growth of ~1 cm to 1 m bodies
  - …
- Turbulence could promote local solid enrichment and local gravitational instability
- The variety of planetary systems (and inside them) could be strongly influenced by the coupling of solid with gas
- **…turbulence does not forbid planet formation!!**

References and cloudy night readings :

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