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The shallow magnitude distribution of asteroid families

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Abstract

It is well known that asteroid families have steeper absolute magnitude (H) distributions for $H < 12$–13 values than the background population. Beyond this threshold, the shapes of the absolute magnitude distributions in the family/background populations are difficult to determine, primarily because both populations are not yet observationally complete. Using a recently generated catalog containing the proper elements of 106,284 main belt asteroids and an innovative approach, we debiased the absolute magnitude distribution of the major asteroid families relative to the local background populations. Our results indicate that the magnitude distributions of asteroid families are generally not steeper than those of the local background populations for $H > 13$ (i.e., roughly for diameters smaller than 10 km). In particular, most families have shallower magnitude distributions than the background in the range 15–17 mag. Thus, we conclude that, contrary to previous speculations, the population of kilometer-size asteroids in the main belt is dominated by background bodies rather than by members of the most prominent asteroid families. We believe this result explains why the Spacewatch, Sloan Digital Sky Survey, and Subaru asteroid surveys all derived a shallow magnitude distribution for the dimmer members of the main belt population.

We speculate on a few dynamical and collisional scenarios that can explain this shallow distribution. One possibility is that the original magnitude distributions of the families (i.e., at the moment of the formation event) were very shallow for H larger than ~ 13, and that most families have not yet had the time to collisionally evolve to the equilibrium magnitude distribution that presumably characterizes the background population. A second possibility is that family members smaller than about 10 km, eroded over time by collisional and dynamical processes, have not yet been repopulated by the break-up of larger family members. For this same reason, the older (and possibly characterized by a weaker impact strength) background population shows a shallow distribution in the range 15–60 km.

Keywords: Asteroids; Collisional physics

1. Introduction

It is now well accepted that the absolute magnitude distributions of the bright members ($H < 12$–13) of asteroid families are very steep (Cellino et al., 1991; Tanga et al., 1999). For most families, these distributions appear to be steeper than the collisional equilibrium distribution computed by Dohnanyi (1969) and the distribution of the background population observed in the same magnitude range. It is evident, however, that these steep distributions cannot hold at all magnitudes or the total mass of the families would be infinite. Collisional evolution studies (Marzari et al., 1995, 1999) suggest that the magnitude distribution of a family has the same slope as the background's distribution for H larger than some threshold value. Marzari's results show that this threshold magnitude value depends on the age of the family in a model-dependent way; currently, there is no observational evidence of what this threshold should be.

Assuming that the steep magnitude distribution of families holds up to $H \sim 18$ (for which both the family and background populations are observationally incomplete), Zappalà and Cellino (1996) argued that asteroid families dominate the overall main belt population, accounting for

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99% of the total number of bodies larger than 1 km. As a consequence, the asteroid belt as a whole should have a steep, family-like, magnitude distribution in the range $13 < H < 18$. In particular, the cumulative magnitude distribution should be of the form $N(< H)\ \alpha\ 10^{0.6H}$. This conjecture has been used in some recent works (Dell'Oro et al., 2001; Tedesco et al., 2002) to make predictions about the collisional evolution of the main belt population and its overall orbital and compositional structure.

The prediction made by Zappalà and Cellino (1996), however, has been challenged by the results of recent main belt asteroid surveys. Jedicke and Metcalfe (1998), debiasing the detections of main belt bodies by the Spacewatch survey, concluded that the exponent of the cumulative magnitude distribution is around 0.3 for $14 < H < 16$ (instead of the 0.6 value predicted by Zappalà and Cellino). More recently, using the detection of $\sim 60{,}000$ main belt asteroids with precise photometry by the Sloan Digital Sky Survey (SDSS), Ivezić et al. (2001) concluded that the cumulative magnitude distribution of the main belt population can be represented by a broken law, with exponent 0.6 only in the range $13 < H < 15$ and exponent 0.25 for $H > 15$. Similarly, the Subaru survey has determined an exponent of 0.2 for the range $16 < H < 19$ (Yoshida et al., 2001).

These results raise the question of what is the real magnitude distribution of asteroid families. In this paper, taking advantage of the June 2002 edition of the catalog of proper elements (106,284 main belt asteroids) by Milani and Knežvić (see Knežvić et al., 2002, for a review), we debias the magnitude distribution of the most prominent asteroid families in the main belt relative to the local background populations in the range $13 < H < 17$ (up to $H < 18$ for the inner belt). We find that the magnitude distributions of the families start to bend around $H = 13$, and that beyond $H = 14$–15, most of them are in fact *shallower* than the distribution of the local background populations. In Section 2 we explain the principle and the method of our debiasing work, while in Section 3 we discuss the results and their implications.

2. Debiasing the distribution of asteroid families relative to the local background populations

The basic principle of our procedure is simple: *a family population and a background population, in the same region of orbital space (semimajor axis* a, *eccentricity* e, *and inclination* i *and in the same absolute magnitude range, should have identical ratios of discovered/undiscovered asteroids.* Note that family members are only determined once their osculating orbital elements have been precisely determined and their proper orbital elements have been computed. Hence, there is nothing in standard observational procedures that would make observers more or less likely to discover a family member than any other main belt asteroid, given similar orbits and absolute magnitudes.

According to this principle, one cannot independently extrapolate the magnitude distributions of an asteroid family and that of the local background population, as in Zappalà and Cellino (1996), because the observational bias (or observational incompleteness) $B(H)$ must be the same in each magnitude bin for the two populations. Instead, given the observed distributions of the two populations, and assuming a debiased distribution for one of the populations, we can compute the debiased distribution of the other population and compare it to the former. In doing so, particular care must be given to define the family and local background population in order to avoid cross-contamination (which would happen, for instance, if most objects of the background population were in reality unidentified family members, or if most of the alleged family members were in reality interlopers from the background population). This operation is delicate, because at the same time we want the family and background population to cover a similar region of orbital element space (in order to justify the assumption that their observational biases are identical). Our procedure is described below.

2.1. Step 1: Identification of asteroid families

To identify asteroid families among the main belt asteroid population, we have followed the method of Zappalà et al. (1995) and have applied a hierarchical clustering method (HCM) to the most recent proper element database of 106,284 numbered and multiopposition main belt asteroids computed by Milani and Knežvić and available through the AstDys website (http://hamilton.dm.unipi.it/astdys). In the HCM method, the differences in proper semimajor axis, eccentricity, and inclination among asteroids in the catalog are translated into differences in orbital velocities using Gauss equations (Morbidelli et al. 1995). Then families are defined by applying a membership criterion that requires that all family members are connected by a "chain," where each member is located within a given velocity difference (cutoff) to its neighbor. The code that performs this automatic classification has been written by one of us (D.N.) using the Zappalà et al. (1995) algorithm and is now freely available from http://www.boulder.swri.edu/~davidn/hcluster. The asteroid families derived by this method strongly depend on the choice of the velocity cutoff: if the latter value is too small, only a handful of objects are identified as family members; if the latter value is too large, the resulting family may incorporate a large share of the entire asteroid population.

It is instructive to look at animations showing how a given family's structure is transformed as the velocity cutoff value is steadily increased (the animations are available on the above-quoted website at SWRI). For the selected families there is a velocity cutoff threshold up to which a quite compact structure is identified and beyond which the entire

Table 1

Family	1st cutoff	No. of members	2nd cutoff	No. of members	No. in background population
Eos	40	1185	90	5188	5184
Hygiea	70	983	110	1703	3181
Koronis	40	1108	90	2663	1611
Themis	60	1425	100	2739	2295
Adeona	50	375	80	648	2510
Dora	50	390	90	419	972
Eunomia	60	2252	110	6162	6963
Gefion	50	682	80	973	2061
Maria	80	1046	110	1776	2039
Flora	70	3021	70	3021	17442
Nys–Pol	60	4744	70	6614	9535
Vesta	50	2840	70	5575	11973

Note. For each of the families considered in this work, the table lists (i) the velocity cutoff used to define its core population (in m/s), (ii) the corresponding number of members, (iii) the velocity cutoff used to define its maximal population, (iv) the corresponding number of members, and (v) the number of observed asteroids in the local background population. "Nys–Pol " stands for the Nysa–Polana family.

local main belt population is incorporated.[1] For many families we choose this threshold to define the *maximal* extent of their populations in the (a, e, i) space, while for others we use a cutoff velocity that is a few tens of meters per second smaller than the threshold, in order to eliminate a peripherical population of bodies that are quite evidently not related to the family in consideration. Moreover, for each family we also choose a significantly smaller velocity cutoff value to define in a conservative way the *core* of the family population. Below, we will use the *core* population to determine the observed H distribution of the family, and the *maximal* family to define, by subtraction (see Section 2.2), the local background (i.e., nonfamily) population. The cutoff values corresponding to the *core* and *maximal* populations for each family considered are listed in Table 1.

2.2. Step 2: Identification of local background populations

By local background of a family, we mean the portion of the main belt population that does not belong to any of the families listed in Table I and that shares (approximately) the same discovery bias of the considered family. For a given absolute magnitude, the discovery bias essentially depends on the sine of the orbital inclination, on the perihelion distance, and on the semimajor axis (Jedicke et al. 2002). To determine the local background population, we first compute the minimal and maximal values of the proper $\sin(i)$, q, and a of the *maximal* population (see Step 1) of the family in consideration, hereafter denoted by $\sin(i)_{\min}$, $\sin(i)_{\max}$, $q_{\min}$, $q_{\max}$, $a_{\min}$, $a_{\max}$. Next, we select all the asteroids in the proper element database with proper $\sin(i)$, q, a satisfying

$$\sin(i)_{\min} - 0.03 < \sin(i) < \sin(i)_{\max} + 0.03 q_{\min} - 0.1 < q < q_{\max} + 0.1 a_{\min} - 0.05 < a < a_{\max} + 0.05, \quad (1)$$

where q and a are expressed in astronomical units. Finally, we subtract from the selected bodies all those that belong to the definition of any of the *maximal* families of Table 1. We call the remaining population the *local background* of the considered family. We apply this operation for each family listed in Table 1, hence defining an equivalent number of local background populations. The number of objects in each background population is listed in the last column of Table 1. The ranges used in (1) have been chosen so that, for most families, the number of bodies defining the local background population is comparable to that of the members of the *maximal* population.

2.3. Step 3: Computation of an empirical bias for the local background

Defining $N_{\rm bg}(H)$ to be the real number of members in the local background population with absolute magnitude between H and $H + dH$ (we use $dH = 1$ in this work), and $n_{\rm bg}(H)$ to be the observed number of members in the same magnitude range, the observational bias (or incompleteness factor) $B(H)$ is defined as

$$B(H) = n_{\rm bg}(H)/N_{\rm bg}(H). \quad (2)$$

According to the principle stated at the beginning of Section 2 we have also

$$B(H) = n_{\rm fam}(H)/N_{\rm fam}(H), \quad (3)$$

where $N_{\rm fam}(H)$ is the real number of members of the family embedded in the local background and $n_{\rm fam}(H)$ is the observed number of members.

[1] See for instance the animation for the Adeona family at http://www.boulder.swri.edu/~davidn/hcluster/adeona.gif. We warn the reader that for most other families the last frame of the animation corresponds to the cut-off velocity threshold.

The local slope of the cumulative magnitude distribution of the background is defined as

$$\alpha_{\mathrm{bg}}(H) = \log\left(\sum_{H'\le H+dH} N_{\mathrm{bg}}(H')\right) - \log\left(\sum_{H'\le H} N_{\mathrm{bg}}(H')\right) \sim N_{\mathrm{bg}}(H + dH)/\sum_{H'\le H} N_{\mathrm{bg}}(H'). \tag{4}$$

Analogously, the local slope of the cumulative magnitude distribution of the family is

$$\alpha_{\mathrm{fam}}(H) = \log\left(\sum_{H'\le H+dH} N_{\mathrm{fam}}(H')\right) - \log\left(\sum_{H'\le H} N_{\mathrm{fam}}(H')\right) \sim N_{\mathrm{fam}}(H + dH)/\sum_{H'\le H} N_{\mathrm{fam}}(H'). \tag{5}$$

Deciding which population (family or background) has a steeper distribution requires computing whether the ratio $\alpha_{\mathrm{bg}}(H)/\alpha_{\mathrm{fam}}(H)$ is larger or smaller than unity. Recalling (2) and (3) one has

$$\frac{\alpha_{\mathrm{bg}}(H)}{\alpha_{\mathrm{fam}}(H)} \sim \frac{n_{\mathrm{bg}}(H + dH)}{n_{\mathrm{fam}}(H + dH)} \times \frac{\sum_{H'\le H}[n_{\mathrm{fam}}(H')/B(H')]}{\sum_{H'\le H}[n_{\mathrm{bg}}(H')/B(H')]}. \tag{6}$$

It is evident that the function $B(H)$ cannot be eliminated from the right-hand side of the equation. This implies that, despite the assumption of equal biases, we cannot conclude on the relative steepness of family and background on the basis of the sole observed distributions. We need to explicitly determine the function $B(H)$.

Unfortunately, the observational bias associated with a given main belt asteroid population is not known a priori because the discoveries were made via a collection of surveys, each having different properties and often operating under non-well-characterized conditions. In absence of a direct knowledge of the bias, we compute an empirical bias for the local background population by assuming that its real magnitude distribution is the same as determined by the SDSS survey for the overall main belt population. In practice, as illustrated in Fig. 1, we define a cumulative distribution function that is equal to that observed for the local background up to $H = 13$ ($H = 14$ if the local background population is in the inner belt and for the local background of the Koronis family). At that point, we follow the slopes determined for the main belt magnitude distribution by the SDSS survey (0.6 for $13 < H < 15$ and 0.25 for $15 < H < 18$). We then identify the associated incremental distribution with the function $N_{\mathrm{bg}}(H)$ in (2), which allows us to compute the bias $B(H)$. Assuming that the standard deviation in the observed differential distribution is Poissonian,

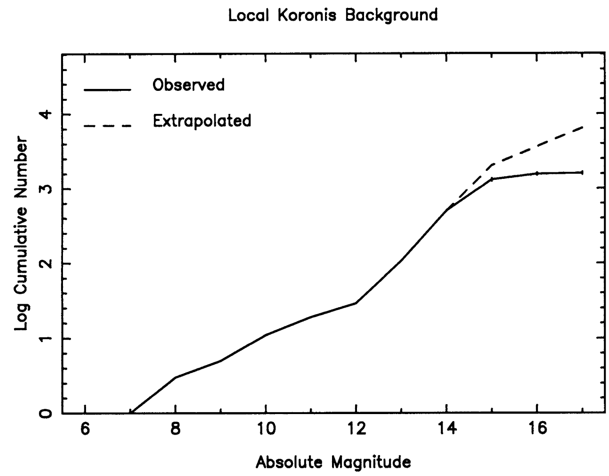

Fig. 1. The observed cumulative magnitude distribution of the asteroids in the local background of the Koronis family (solid) and an extrapolation obtained assuming that the slopes of the real distribution are those determined by the SDSS survey for the overall main belt population with $H > 13$ (dashed). The error bars on the observed distribution are computed assuming that the counts of asteroids in each magnitude bin follow Poissonian statistics (see formula (7)).

$$\sigma_{n\mathrm{bg}}(H) = \sqrt{n_{\mathrm{bg}}(H)}, \tag{7}$$

the standard deviation of the bias function is

$$\frac{\sigma_B(H)}{B} = \frac{\sigma_{n\mathrm{bg}}(H)}{n_{\mathrm{bg}}(H)} = \frac{1}{\sqrt{n_{\mathrm{bg}}(H)}} \tag{8}$$

(we do not consider errors associated with $N_{\mathrm{bg}}(H)$ because the latter at this stage is an *assumed* function, and not a function computed from data). In the magnitude range where the population is considered observationally complete ($H < 13$ or $H < 14$), $B(H) = 1$ and we assume that $\sigma_B(H) = 0$.

The assumption that the true magnitude distribution of the local background population follows the slopes determined by SDSS is arbitrary, but it is motivated by the consideration that the background population, if it is in collisional equilibrium, should have the same approximate distribution everywhere. Note that the assumed distribution in Fig. 1 extrapolates very nicely the observed distribution in the range $13 < H < 14$, supporting the idea that the SDSS slope(s) should be applicable also to the local background(s). As it will become clear in Section 2.4, the choice of the "true " background distribution determines the value of the bias function, and hence that of the family's debiased magnitude distribution. The ratio (6) between the slopes, however, only moderately depends on the value of the bias function. This is because the sums on the right-hand side of

(6) are essentially dominated by the last terms $n_{\rm fam}(H)/B(H)$ and $n_{\rm bg}(H)/B(H)$. Hence $B(H)$ can (approximately) be factorized and eliminated. As a consequence, the exact choice of the background distribution will *not* affect our conclusions on the steepness of the family magnitude distributions relative to the local backgrounds.

2.4. Step 4: Computation of debiased family distribution

Using our value of $B(H)$ and inverting Eq. (3), we can compute the debiased incremental distribution of the family $N_{\rm fam}(H)$. The debiased cumulative distribution is then

$$\bar{N}_{\rm fam}(H) = \sum_{H' \le H} N_{\rm fam}(H'). \tag{9}$$

In the incompleteness region ($H > 13$ or $H > 14$), we also assume that the observed incremental distribution of the family has a Poissonian standard deviation:

$$\sigma_{n_{\rm fam}}(H) = \sqrt{n_{\rm fam}(H)}\ . \tag{10}$$

Then, using (7), (10), and the standard formulæ for error propagation, we obtain that the standard deviation of $N_{\rm fam}(H)$ is

$$\sigma_{N_{\rm fam}}(H) = N_{\rm fam}(H) \times \sqrt{\frac{1}{n_{\rm fam}(H)} + \frac{1}{n_{\rm bg}(H)}} \tag{11}$$

for $H > 13$ or $H > 14$ (and 0 otherwise) and the standard deviation of the debiased cumulative distribution (9) is

$$\sigma_{\bar{N}_{\rm fam}}(H) = \sqrt{\sum_{H' \le H} \sigma^2_{N_{\rm fam}}(H')}. \tag{12}$$

Fig. 2 shows the observed and debiased H distribution of the Koronis family, with error bars corresponding to the standard deviations computed with the above recipe. A comparison with the slopes assumed for the local background indicates that the family's magnitude distribution is shallower than that of the background for $H > 14$ (the comparison with Fig. 1 shows that the family is shallower than the background starting from $H = 12$). The limited extent of the error bars implies that this difference is statistically significant.

We have tried several different assumed magnitude distributions for the local background: single-slope distributions, two-slope distributions, steep distributions, and shallow distributions. In all cases, we have obtained the same result: the debiased magnitude distribution of the Koronis family is shallower than the distribution assumed for the background. Given the discussion at the end of Section 2.3, this result should not be a surprise. The assumption that the bias function $B(H)$ is the same for family and background makes the two distributions correlated. As the assumed background distribution becomes shallower, $B(H)$ becomes smaller and the resulting debiased family distribution becomes shallower. Therefore, the resulting shape and slopes of the debiased family distribution can be questioned—because they depend on what was assumed as the true distribution for the background (although we believe that our choice, based on the SDSS results, is reasonable)—but *not* the general result that the family distribution is shallower than the background distribution.

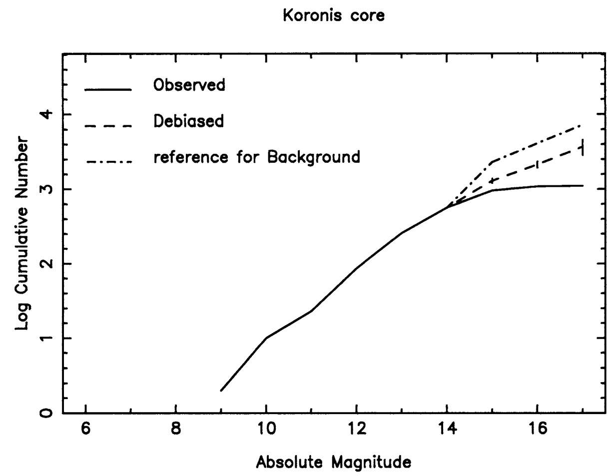

Fig. 2. The observed cumulative magnitude distribution of asteroids in the *core* of the Koronis family (solid) and the extrapolation obtained assuming that the observational bias is the same as for the local background (dashed). The error bars on the debiased distribution are computed using (12). For comparison, the SDSS slopes assumed for the background are also plotted as dashed-dotted lines, showing that the magnitude distribution of the Koronis family is shallower than that of the local background for $H > 13$.

3. Results for the main families and discussion

The left panels of Fig. 3 show the debiased cumulative distributions (with error bars) that we have obtained for the *cores* of all the families listed in Table 1. From top to bottom, the panels illustrate the results for the families in the outer belt ($a > 2.8$ AU), central belt ($2.5 < a < 2.8$ AU), and inner belt ($a < 2.5$ AU). The cumulative distribution for the overall background population in the considered portion of the belt is also plotted for reference. The slopes assumed for the background population beyond $H = 13$ ($H = 14$ in the inner belt) are those determined by SDSS, and are the same as we have used for the definition of the *local* background distributions of all families.

All families have a cumulative distribution that is not steeper than that of the background beyond $H = 13$ (with the exception of marginal cases such as the Vesta and Nysa–Polana families in the range $13 < H < 15$ and the Maria family in the range $16 < H < 17$). This explains why the asteroid surveys (Spacewatch, SDSS, Subaru) detected a shallow magnitude distribution for the overall main belt population. Had the steep family distributions continued beyond $H = 13$, the overall main belt population would have been dominated by family members, and therefore would have shown a steep (family-like) magnitude distribution, as expected in Zappalà and Cellino (1996).

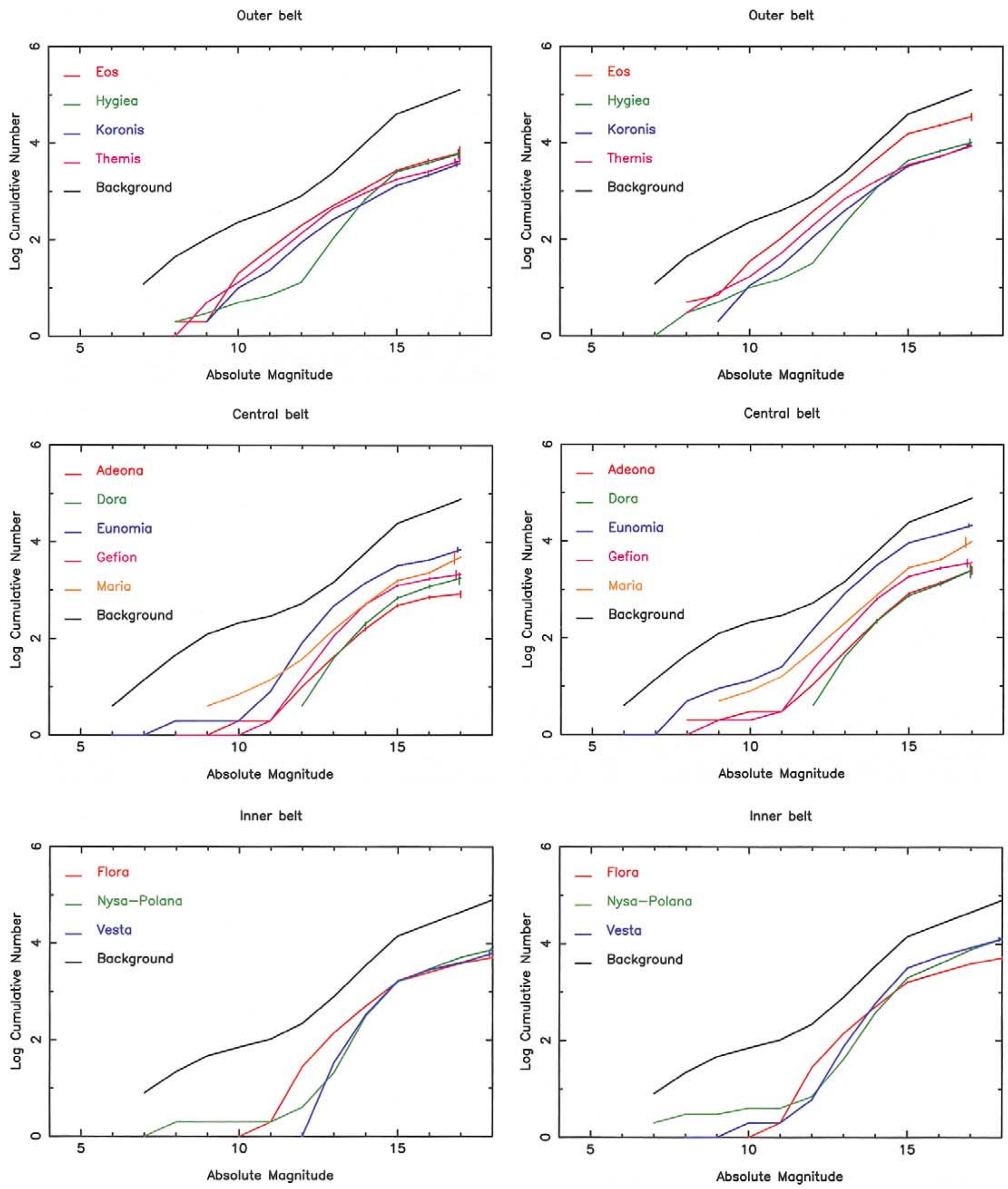

Fig. 3. (Left) The debiased cumulative magnitude distribution of the asteroids in the *core* of the most prominent families of the asteroid belt (colored curves). The corresponding standard deviations are indicated by vertical bars (often invisible because of their shortness) with the same colors. The black curves show the background distribution in the three main belt zones, assuming the slopes determined by the SDSS survey for $H > 13$ ($H > 14$ for the inner belt). (Right) The same, but plotting the debiased cumulative magnitude distribution of the asteroids in the *maximal* family populations.

Our conclusion that most families have a magnitude distribution *shallower* than that of the background is surprising, given the widespread view in the field that family distributions should be steeper than the background. Before entering into a discussion of why it can be so, we first review and debate all possible reasons to believe that our result might be an artifact:

(i) As we acknowledged at the beginning of this paper, the debiasing procedure is delicate and relies on the fundamental assumption that the local background population and the family population suffer the same observational biases. Although we center the local background population around the family population in the space of the fundamental orbital parameters that govern the discovery probability ($\sin(i)$, q, and a), it is possible that the mean values of these elements for the background and the family are somewhat different, so that the mean biases for the two populations are not exactly the same. We concede this potential difference in bias may occur for some specific families, but Fig. 3 shows a systematic result. To create a situation where all families have cumulative magnitude distributions steeper than the background, we would need the observational completeness for each family to be smaller (i.e., the bias is more severe) than that of the corresponding background populations. We see no reason why this should be true. In fact, for the Themis family, we are almost certain that the bias is less severe than for the background because the family extends down to proper $i = 0$; this allows the velocity cutoff criterion (1) to select background bodies that have, on average, a larger proper inclination, making them more difficult to detect than Themis family members. Even in this extreme case, the debiased magnitude distribution of the Themis family appears significantly shallower than that of the local background.

(ii) The local background population might be dominated by unidentified family members, such that what we assume as background could, in fact, be a part of the family. If this were true, the observed and debiased distributions for family/background would be essentially identical. The fact that most families appear to be shallower beyond $H = 15$ implies that what we observe in the local background and in the *core* of the family are not representative of the same population.

(iii) It is possible that small asteroids have poorly known orbits, yielding proper elements with large enough errors that they cannot be identified as family members. In this circumstance, the family would be artificially depleted of small bodies. To test this hypothesis, for the most prominent families, we restricted our analysis to a proper elements catalog of 23,845 numbered asteroids. Our results showed the same trend for families relative to the background, although the errors were somewhat larger because of the reduced quantity of data.

(iv) Small family members are more dispersed in proper element space than large members (Cellino et al., 1999), possibly because of their original ejection velocity field (Michel et al., 2001) and/or the subsequent size-dependent mobility of objects due to the Yarkovsky effect (Bottke et al., 2001). As a consequence, many small family members may be missing from the definition of the *core* population, making the magnitude distribution artificially shallow. To test this hypothesis, for each family, we repeated our analysis using the *maximal* population (defined with the largest possible cutoff) instead of the *core* population. The resulting debiased distributions are shown in the panels reported on the right-hand side of Fig. 3. In comparison with the panels on the left-hand side, we remark that—although a much larger number of members are now included in the families—the shapes of the cumulative magnitude distributions have not significantly changed. Most of the families continue to have cumulative magnitude distributions shallower than the background distribution. The only notable exceptions are those of the Adeona and the Nysa–Polana families, for which the magnitude distributions of the *core* populations are shallower than the background, but those of the *maximal* populations have approximately the background slope. None of the 12 families we considered has a cumulative distribution significantly steeper than the background distribution. Therefore, we conclude that the precise value of the cutoff in the allowed range for each family does not change the significance of our result.

Having concluded that the shallow magnitude distribution of asteroid families is not an artifact of our debiasing procedure, we now speculate on its possible origin.

It is now generally accepted, from the observation of asteroid families (Tanga et al., 1999) and numerical experiments (Michel et al., 2001), that the break-up of a parent body generates an ensemble of fragments with a resulting steep size (or magnitude) distribution. The slope of this distribution, however, cannot be extrapolated to arbitrary small sizes or the total mass of the fragments would exceed that of the parent body. The break-up of the parent body should therefore produce a two-slope distribution, with a steep slope for the large fragments and a shallower slope for the small fragments (see Asphaug and Melosh, 1993 and experiments in Davis and Ryan, 1990). It is unclear, however, at which sizes the distribution becomes shallower, and simulations by Michel et al. do not yet have the resolution needed to provide an answer to this question. Marzari et al. (1999), who modeled break-up events with two slopes in their collisional evolution simulations, found that the best match to the observed population was obtained with an inflection point in the size distribution at about 1–10 km in diameter. For families generated by bodies approximately 100 km in diameter, this means that the inflection point in the fragment size distribution occurs at sizes that are at least an order of magnitude smaller than that of the parent body.

On the other hand, background bodies with $H > 13$ should also be asteroid fragments because their collisional lifetimes are too short (of order 1 Gy; Farinella and Davis, 1992; Bottke et al., 1994) for them to be primordial objects. Therefore, the background population at small sizes may be a collection produced by a multitude of families. These families have not yet been individually recognized, presumably because their size distributions start with 5–10 km diameter objects, so that only a handful of members have been observed so far in each group. Because these small families should have been generated by parent bodies much smaller than those of the major families listed in Table 1 ($\sim$15 km, instead of $\sim$100 km) the size distribution of the generated fragments could be steep over the multikilometer size range and shallow for subkilometer bodies. If true, this could explain why the magnitude distribution of the overall background population looks steeper than that of the major families in the range $13 < H < 18$. Eventually, collisional evolution (e.g., Marzari et al., 1995, 1999) of both background and major families should smooth this difference, but the timescale needed to erase it completely is unknown. If the conjecture is correct, this timescale would be longer than the age of the asteroid belt.

On the other hand, if the original size distributions of the major families were steep all the way to subkilometer bodies, their currently observed shallow distributions imply that many— or most— of the family members in the range 13

$< H < 18$ were eliminated over the ages of the families. The elimination of small bodies could be due, in principle, to dynamical and collisional processes. Bottke et al. (2001) showed that small family members drift away from the family center due to the Yarkovsky effect, so that some are captured into powerful resonances, and are dynamically ejected from the asteroid belt. We doubt, however, that this process could significantly alter the magnitude distribution of asteroid families. For example, to obtain that $\bar{N}_{\rm fam}(17)/\bar{N}_{\rm fam}(13)$ is equal to $\bar{N}_{\rm bg}(17)/\bar{N}_{\rm bg}(13)$ ($\bar{N}_{\rm pop}(H)$ denoting the number of bodies with absolute magnitude brighter than H in the considered population), the population of the *core* of the Koronis family in the range $13 < H < 17$ should be multiplied by a factor of 3.7, and that of the *maximal* family by a factor of 2.3. If the shallow magnitude distribution of the family were due to the dynamical elimination of the objects, this would imply that from 56 to 73% of the original bodies from 1 to 10 km in diameter have been eliminated. This seems too much even in the most optimistic models of the Yarkovsky effect.

The collisional elimination of small family members may be a more promising process. Durda and Dermott (1997) have pointed out that "asteroid families with material properties (i.e., strength scaling laws) that differ from that of the average background population may evolve a size distribution with a different equilibrium slope than that of the background." We elaborate here on this idea. Fig. 3 shows the background population has a wavy magnitude distribution. These waves have been interpreted by Campo Bagatin et al. (1994) and Durda et al. (1998) using collisional evolution models. In essence, these model predict that the shallow slope in the region $9 < H < 12$ is due to the fact that many bodies in this range were collisionally disrupted before they could be significantly replenished by the break-up of larger bodies ($H < 9$). In this case, the larger bodies may be too difficult to disrupt (and when they break they generate a major asteroid family and hence their products are not counted in the background population). Therefore, the background population is strongly depleted in $9 < H < 12$ bodies. In turn, the background distribution in the range $12 < H < 15$ is steep again, because these bodies here have been produced in large number by the break-up of the $9 < H < 12$ asteroids.

When we look at the major asteroid families (Fig. 3), we see that the distribution in the range $9 < H < 12$ is steep. This suggests that family members in this range have not undergone disruption events at nearly the same level as those bodies in the background population. If true, this result would suggest that families are younger than the background population (Marzari et al., 1995, 1999) and/or large family members may have an increased impact strength relative to primordial asteroids of the same size. One way to explain the latter would be to make family members aggregates of small fragments—as suggested by Michel et al. (2001) simulations—while giving primordial bodies significantly different internal structures and/or physical properties. Another would be to assume that the shape of the background population is not a by-product of collisional evolution in the present-day main belt, but rather a fossil of disruption events that occurred in a massive, primordial main belt. Whatever the reason, the paucity of fragmentation events among family members with $H < 12$ has failed to generate a large number of second generation family members with $H > 12$. Consequently, this latter population has been eroded away by collisional grinding but has not yet been regenerated by collisional cascades initiated by larger bodies. In other words, the families reproduce, at fainter magnitudes, the wave that we observe in the background distribution in the range $9 < H < 12$.

Although we hypothesize that this collisional process could explain why the current H distribution of families is so shallow for faint bodies, we cannot, as of yet, offer any proof. Quantitative simulations of the collisional evolution of both asteroid families and background populations will be needed to support or reject this hypothesis, both beyond the scope of this work.

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